

# Broadband Rational Modeling using Barycentric Vector Fitting

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## Abstract

This paper demonstrates the use of a Barycentric Vector Fitting algorithm for the modeling of broadband frequency responses. This algorithm reduces the computation time of the pole-identification step by exploiting the specific form of the barycentric interpolation formula. Numerical results confirm that this approach can lead to significant savings in terms of computation time, while preserving a good accuracy of the model. The effectiveness of the algorithm is shown by applying it to a 4-port board-to-board interconnect.

## 1 Introduction

The identification of broadband models from tabulated frequency responses plays a crucial role in the accurate simulation of microwave components and systems. In the literature, it was shown that Vector Fitting (VF) [1] is a highly robust method to calculate such models, and its use has been adopted in many domains of applied engineering, including power systems, packaging and electromagnetic compatibility. The algorithm has been studied intensively and many extensions, improvements and enhancements have been reported. Some of them include the relaxation of the model coefficients, a fast QR step, orthonormalization of the basis functions, z-domain transformations, enhanced robustness to noise, parameterization schemes, time-domain fitting, passivity enforcement and many others,.... A comprehensive survey on frequency domain identification using VF has been reported in [2] and the references therein.

A novel development is the Barycentric Vector Fitting algorithm which aims to further reduce the computation times, both in the case of single-port or multi-port systems. All the details about the algorithm can be found in [3]. It is shown in this paper that the method is able to compute an accurate macromodel for a board-to-board interconnect with reduced CPU time.

## 2 Overview of Algorithm

The goal of the barycentric vector fitting algorithm is to compute a macromodel that approximates all scattering elements  $H_v(s)$  (for  $v = 1, \dots, V$ ) of a transfer matrix with a common set of  $P$  poles over a given frequency range of interest  $[s_{\min}, s_{\max}]$ . The main steps of the algorithm can be outlined as follows :

- First, the system under study is simulated or measured over the frequency range and all the S-parameter data samples are split in 2 disjoint sets  $\bar{S}$  and  $S$ , such that  $\bar{S} \cap S = \phi$ .

$$\bar{S} = \{\bar{s}_m, H_v(\bar{s}_m)\}_{m=1}^M, S = \{s_k, H_v(s_k)\}_{k=1}^K \quad (1)$$

The set  $\bar{S}$  contains a small amount of  $M=P/2$  uniformly spaced samples [1] and its size is chosen as a function of the desired model complexity. The set  $S$  comprises all the  $K$  remaining data samples (where usually  $K \gg M$ ).

- Then, the barycentric interpolation formula is applied to compute a rational model that interpolates the data in  $\bar{S}$ .

$$R_v(s) = \frac{N_v(s)}{D(s)} = \frac{\sum_{m=1}^M \frac{w_m}{s - \bar{s}_m} H_v(\bar{s}_m)}{w_0 + \sum_{m=1}^M \frac{w_m}{s - \bar{s}_m}} \quad (2)$$

A linear combination of the basis functions is formed to ensure that complex conjugacy of the poles and zeros is enforced analytically [3]. A key advantage of interpolation formula (2) is that the choice of barycentric weights provides flexibility to control the model behavior inbetween the samples  $\bar{S}$ . Here, these weights  $w_m$  (i.e. model coefficients) are computed using an iterative least-squares approach to approximate the remaining samples  $S$  in a least-squares sense. When compared to the partial fraction expansion in the standard VF, it is seen that all numerators  $N_v(s)$  and denominator  $D(s)$  share the same set of unknowns. This leads to a smaller set of LS equations that can be solved in a reduced amount of computation time.

- In order to compute the values of the barycentric weights  $w_m$ , it is possible to use a Sanathanan-Koerner (SK) iteration with explicit weighting as detailed in [2]. First, the formula (2) is linearized by multiplying both sides of the equation with  $D(s)$ , and a first estimate of the model coefficients is found by solving a set of linear equations. In order to relieve the bias of the linearization step, the SK iteration is applied to iteratively compute updated values of the model coefficients. The relocated poles of the transfer function  $R_v(s)$  are found by solving an eigenvalue problem, and stability of the poles can be enforced, as in [1].
- Once the poles of the transfer function  $R_v(s)$  are obtained, it is possible to apply the residue identification procedure of the standard VF algorithm to compute a model that approximates all the data (i.e.  $\bar{S} \cup S$ ) in a least-squares sense. This yields a model in the partial fraction form which can directly be realized as a set of state-space equations [1].

## 3 Example : Measured board-to-board interconnect

This example demonstrates the modeling of a 32 inch board-to-board interconnect system with two HM-Zd connectors in FR4, whose S-parameter response is measured at 800 frequencies over the range of interest [0.05 GHz – 20 GHz]. The measurement setup is shown in Fig. 1. All 16 elements of the (noisy) scattering matrix are modeled using a common set of  $P = 400$  poles using the Barycentric Vector Fitting algorithm and the result is shown in Fig. 2. It is clear that the model shows a good agreement with the reference data. In Fig. 3, the

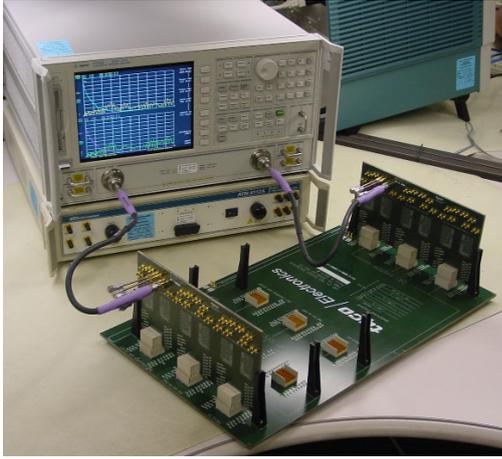


Figure 1: Measurement setup

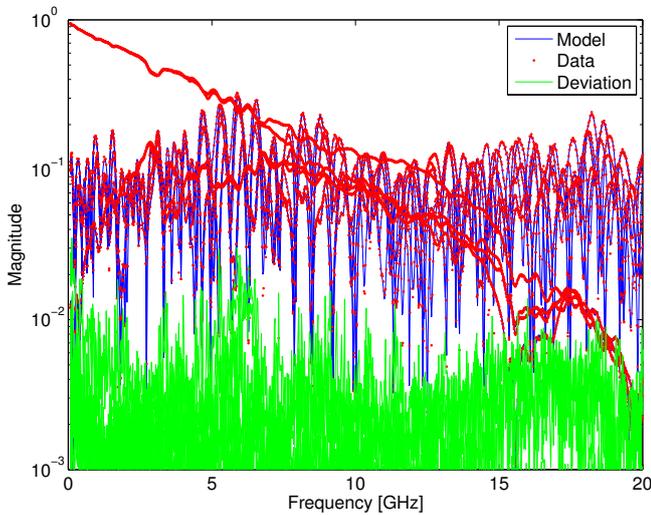


Figure 2: Magnitude of data and model.

evolution of the RMS error is visualized as a function of the iteration count, and it is seen that the accuracy and convergence is comparable to the Fast QR-based VF algorithm [1][4][5] that is publicly available at [6]. An important point is that the BVF algorithm was able to compute the final macromodel in only 12.5 seconds, whereas the Fast QR-based VF algorithm takes 28.8 seconds. (Note that all timing results are performed in MATLAB on a 64-bit operating system with Intel(R) Core(TM) i7-2760 QM CPU at 2.4 GHz and 8 GB RAM. The starting poles for Fast QR implementation are chosen in an optimal way as discussed in [1])

#### 4 Conclusions

A Barycentric Vector Fitting algorithm was applied for the macromodeling of frequency responses. It exploits the barycentric interpolation formula in order to speed-up the pole identification step. Numerical results show that the method delivers accurate results, even when the data is contaminated with noise. It can improve the running time of existing VF algorithms when fitting of single/multiple elements of a transfer matrix.

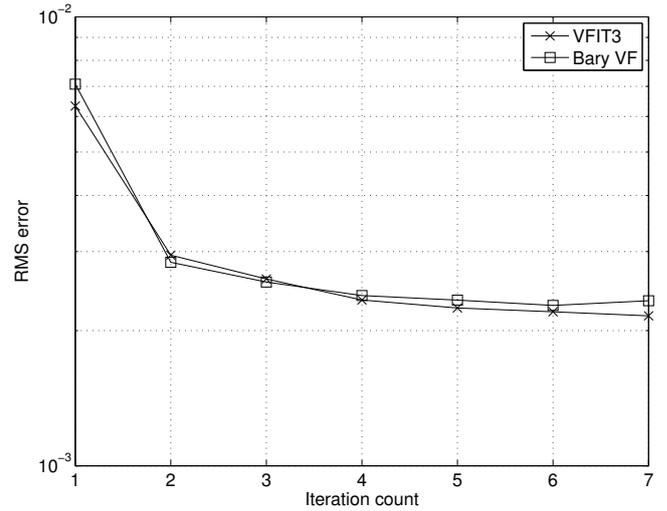


Figure 3: Evolution of RMS error vs. iteration count.

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