

Reduced Order Modeling of delayed PEEC circuits

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Abstract—We propose a novel model order reduction technique that is able to accurately reduce electrically large systems with delay elements, which can be described by means of neutral delayed differential equations. It is based on an adaptive multipoint expansion and model order reduction of equivalent first order systems. The neutral delayed differential formulation is preserved in the reduced model. Pertinent numerical results validate the proposed model order reduction approach.

Index Terms—Delayed Partial Element Equivalent Circuit method, reduced order systems, neutral delayed differential equations.

I. INTRODUCTION

With the increase of operating frequencies, the accurate modeling of high-speed components and modern integrated circuits requires electromagnetic (EM) 3-D methods [1], [2] as necessary analysis and design tools. Large systems of equations are usually generated by the use of these methods and model order reduction (MOR) techniques are crucial to reduce the complexity of EM models and the computational cost of the simulations, while retaining the important physical features of the original system [3], [4]. Among all EM methods, the Partial Element Equivalent Circuit (PEEC) method [2] uses a circuit interpretation of the Electric Field Integral Equation (EFIE) [5], therefore it is especially suitable to problems involving both electromagnetic fields and circuits [2], [6]. If we consider a quasi-static PEEC formulation [2], which approximates the full-wave PEEC approach [7], an equivalent RLC circuit is obtained, since the time delays between the elements in the full-wave PEEC formulation are neglected. Systems of ordinary differential equations (ODE) are obtained. Standard MOR techniques for ODE systems can be used to reduce the size of a quasi-static PEEC model [3], [4]. When signal waveform rise times decrease and the corresponding frequency content increases or the geometric dimensions become electrically large, time delays must be taken into account and, therefore, included in the modeling.

A PEEC formulation which include delay elements, called τ PEEC [8], becomes necessary and leads to systems of neutral delayed differential equations (NDDE) [7]. While several successful MOR methods for large ODE systems have been proposed over the years, the reduction of large NDDE systems is still a very challenging research topic, since standard MOR techniques for ODE systems cannot be directly applied to NDDE systems. Especially, the reduction of electrically large structures where delays among coupled elements cannot be

neglected or easily approximated by rational basis functions needs to be investigated and addressed.

Some techniques for the reduction of NDDE systems have been proposed [9]–[12]. In [9] an equivalent first order system is computed by means of a Taylor expansion, and then MOR Krylov subspace methods [3], [4] are applied. The NDDE formulation is not preserved in the reduction process. Multiple expansion points and the construction of rational local approximations of τ PEEC systems are used in [10]. Then, each rational local approximation is reduced by standard MOR methods and portions of these reduced rational models are combined to obtain a global reduced model for the frequency range of interest. As in [9], the NDDE formulation is not preserved in the reduction process. The approach in [11] extracts exponential terms (primary phase factors) and then the smoother remainders are expanded into a linear form and then projected to obtain the reduced model. Hence, the extraction of primary phase factors and the segregation of the system into multiple remainder phase matrices, each corresponding to a primary phase factor, are needed. In [12], an equivalent first order system is computed by means of a single point Taylor expansion [9] and a corresponding orthogonal projection matrix is computed by means of a block Arnoldi algorithm [3]. Then, an orthogonal projection matrix for the original NDDE system is extracted and a reduced NDDE system is obtained.

This paper presents a novel MOR method for τ PEEC models that is able to accurately reduce electrically large structures where delays among coupled elements cannot be neglected or easily approximated by rational basis functions. It is based on an adaptive multipoint expansion and MOR of equivalent first order systems [12]. The NDDE formulation is preserved in the reduced model.

The paper is organized as follows. Section II describes the modified nodal analysis (MNA) equations of the τ PEEC method. Section III describes the proposed MOR method for NDDE systems. Finally, a pertinent numerical example based on the τ PEEC method validates the proposed technique in Section IV.

II. DELAYED PEEC FORMULATION

The PEEC method [2] stems from the integral equation form of Maxwell's equations. With respect to other EM methods, it is worth pointing out its capability to provide a circuit interpretation of the EFIE equation, thus allowing to handle

Algorithm 1: MULTIPPOINT($NDDE_{orig}, \{s_k\}_{k=1}^{n_{points}}, n_r, q$)

comment: Multipoint computation of basis \mathbf{Q}

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 $\mathbf{Q}_1 = \text{One point DMOR}(NDDE_{orig}, s_1, n_r, q);$ 
 $\mathbf{Q} = \mathbf{Q}_1;$ 
for  $k \leftarrow 2$  to  $n_{points}$ 
  do  $\begin{cases} \mathbf{Q}_k = \text{One point DMOR}(NDDE_{orig}, s_k, n_r, q); \\ \mathbf{Q} = ([\mathbf{Q}, \mathbf{Q}_k]); \end{cases}$ 
 $\mathbf{Q} = \text{orth}(\mathbf{Q});$ 

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Fig. 2. Pseudocode for computing the orthogonal basis \mathbf{Q} in the multipoint expansion case.

$$s\mathbf{C}_r(s)\chi(s) = -\mathbf{G}_r(s)\chi(s) + \mathbf{B}_r\mathbf{V}_p(s) \quad (6)$$

$$\mathbf{I}_p(s) = \mathbf{B}_r^T \chi(s) \quad (7)$$

$$\mathbf{C}_r(s) = \mathbf{C}_{r,0} + \sum_{k=1}^{n_\tau} \mathbf{C}_{r,k} e^{-s\tau_k} \quad (8)$$

$$\mathbf{G}_r(s) = \mathbf{G}_{r,0} + \sum_{k=1}^{n_\tau} \mathbf{G}_{r,k} e^{-s\tau_k} \quad (9)$$

is obtained, where the following congruence transformations are used

$$\mathbf{C}_{r,i} = \mathbf{Q}^T \mathbf{C}_i \mathbf{Q}, \quad i = 0, \dots, n_\tau \quad (10a)$$

$$\mathbf{G}_{r,i} = \mathbf{Q}^T \mathbf{G}_i \mathbf{Q}, \quad i = 0, \dots, n_\tau \quad (10b)$$

$$\mathbf{B}_r = \mathbf{Q}^T \mathbf{B} \quad (10c)$$

$$\mathbf{L}_r = \mathbf{Q}^T \mathbf{L} \quad (10d)$$

IV. NUMERICAL RESULTS

Concerning the error criterion, let us define the weighted RMS-error as

$$\begin{aligned}
 Err &= \\
 &= \sqrt{\frac{\sum_{i=1}^{(n_p)^2} \sum_{k=1}^{K_s} |w_{Y_i}(s_k) (Y_{r,i}(s_k) - Y_i(s_k))|^2}{(n_p)^2 K_s}} \quad (11)
 \end{aligned}$$

with

$$w_{Y_i}(s) = |(Y_i(s))^{-1}| \quad (12)$$

It is used to assess the accuracy of reduced order models over the bandwidth of interest, which is sampled over K_s samples.

A. Multiconductor system

A multiconductor system composed of four coplanar conductors with a length $\ell = 10$ cm, a width $W = 1$ mm, a thickness $t = 1$ mm and an horizontal spacing $S_x = 3$ mm has been modeled in this example. Fig. 3 shows its cross section. The system is analyzed on the frequency range $[0.0001, 10]$ GHz.

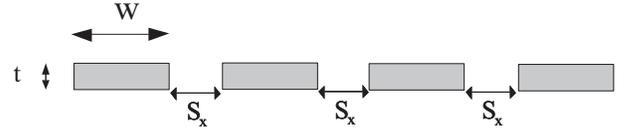


Fig. 3. Cross section of the multiconductor system.

The six ports of the system are defined between successive conductors.

The order of the original τ PEEC model is equal to $n_u = 2390$, the number of delays is equal to 632 and the largest delay gives $2\pi \text{freq}_{max} \tau_{max} = 21$. Table I shows some parameters of the proposed MOR method.

TABLE I
MOR PARAMETERS.

Parameter	Adaptive method
Taylor terms	2
Expansion points	7
Reduced order	84
Weighted RMS-error reduced model	0.025

Figs. 4-7 compare the magnitude and phase of the original and reduced NDDE model of $\mathbf{Y}_{33}(s)$ and $\mathbf{Y}_{36}(s)$. As clearly seen, the presented MOR method can reduce a large NDDE system with large delay terms, while accurately preserving the behavior of the original system.

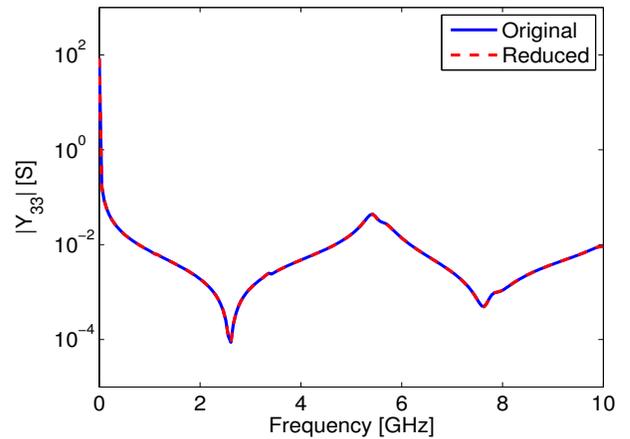


Fig. 4. Magnitude of \mathbf{Y}_{33} .

V. CONCLUSIONS

We have presented a new MOR technique for large NDDE systems, which is applicable to τ PEEC models. It is able to accurately reduce electrically large structures where delays among coupled elements cannot be neglected or easily approximated by rational basis functions. It is based on an adaptive multipoint expansion and MOR of equivalent first order systems. The NDDE formulation is preserved in the

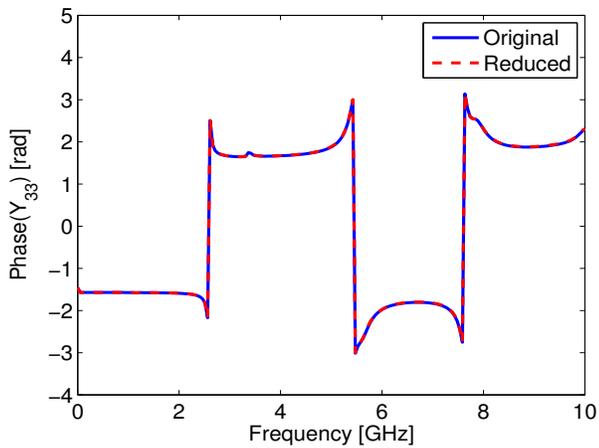


Fig. 5. Phase of Y_{33} .

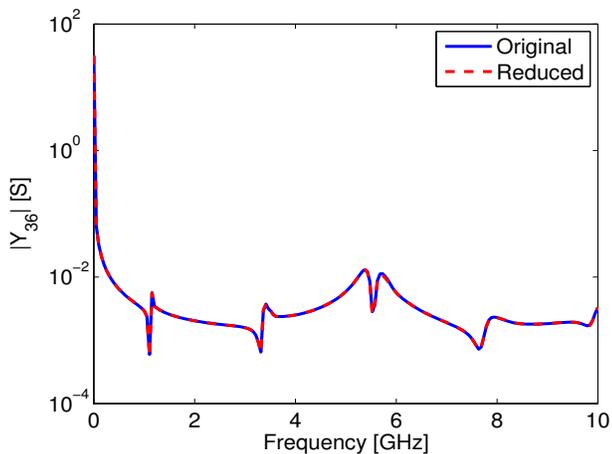


Fig. 6. Magnitude of Y_{36} .

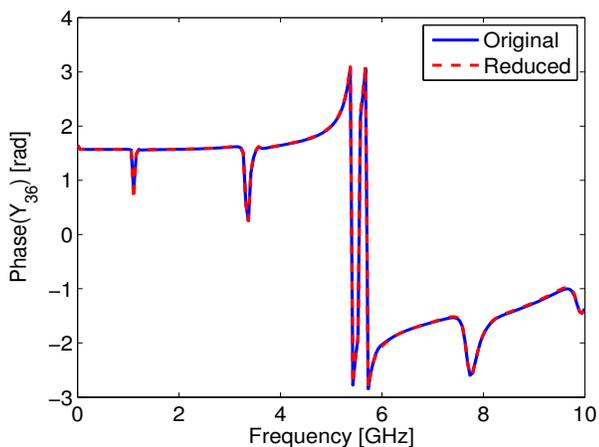


Fig. 7. Phase of Y_{36} .

reduced model. Numerical results based on the τ PEEC method

have validated the proposed MOR approach.

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