Parameterized S-Parameter Based Macromodeling With Guaranteed Passivity

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***Abstract*—This letter presents a novel parametric macromodeling technique for scattering input-output representations parameterized by design variables such as geometrical layout or substrate features. It provides accurate multivariate macromodels that are stable and passive by construction over the entire design space. Overall stability and passivity of the parametric macromodel are guaranteed by an efficient and reliable combination of rational identification and interpolation schemes based on a class of positive interpolation operators.**

***Index Terms*—Interpolation, parametric macromodeling, passivity, rational approximation.**

I. Introduction

ROBUST parametric macromodeling is becoming increasingly important for efficient design space exploration, design optimization and sensitivity analysis of microwave structures. Parametric macromodels can take multiple design variables into account, such as geometrical layout or substrate features.

Recently, a multivariate extension of the orthonormal vector fitting (OVF) technique was presented in [1]. It was shown that this approximation method accurately models parameterized frequency responses with a highly dynamic behavior. Unfortunately, the algorithm does not guarantee stability and passivity. A solution to the stability problem is presented in [2], where a parametric macromodel is computed with barycentric interpolation of univariate stable macromodels. Overall stability of the macromodel is guaranteed by construction and a passivity enforcement scheme is proposed by perturbation of the barycentric weights. However the passivity violations must be reasonably small and the convergence of the passivity enforcement procedure is not guaranteed. The technique proposed in [3] can handle scattered data and takes advantage of the flexibility of least-squares fitting, while preserving stability. More recently, a novel technique that combines the advantages of [1] and [3] was presented in [4]. The hybrid technique is able to calculate more compact macromodels without compromising the accuracy of the results. It is less sensitive to the sample density and the overall stability of the poles is preserved.

This letter presents a novel technique to build accurate multivariate rational macromodels for scattering representations that are stable and passive over the entire design space. The technique is validated by a numerical example.

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II. Parametric Macromodeling

The goal of the proposed algorithm is to build a multivariate representation $\mathbf{R}(s,\vec{g})$ which accurately models a large set of K_{tot} data samples $\{(s,\vec{g})_k,\mathbf{H}(s,\vec{g})_k\}_{k=1}^{K_{tot}}$ and guarantees stability and passivity over the entire design space. These data samples depend on the complex frequency $s=j\omega$, and several design variables $\vec{g}=(g^{(n)})_{n=1}^{N}$, such as the layout features of a circuit (e.g., lengths, widths,...) or the substrate parameters (e.g., thickness, dielectric constant, losses,...).

A. Root Macromodels

Starting from a set of data samples $\{(s,\vec{g})_k,\mathbf{H}(s,\vec{g})_k\}_{k=1}^{K_{tot}}$ a frequency dependent rational model in a pole-residue form is built for all grid points in the design space by means of Vector Fitting (VF) technique [5]. A pole-flipping scheme is used to enforce strict stability [5] and passivity enforcement can be accomplished using one of the robust standard techniques [6]–[8]. The result of this initial step is a set of rational univariate macromodels, stable and passive, that we call *root macromodels* being the starting points to build the global parametric macromodel.

B. 2-D Macromodeling

First, we discuss the representation of a bivariate macromodel and afterwards the generalization to more dimensions. Once the *root macromodels* are available, the next step is to find a bivariate representation $\mathbf{R}(s,g)$ which models the set of K_{tot} data samples $\{(s,g)_k,\mathbf{H}(s,g)_k\}_{k=1}^{K_{tot}}$ and preserves stability and passivity over the entire design space. The bivariate macromodel we adopt can be written as

$$\mathbf{R}(s,g)=\sum_{k=1}^{K_1}\mathbf{R}(s,g_k)\ell_k(g) \tag{1}$$

where the interpolation kernels $\ell_k(g)$ are scalar functions satisfying the following constraints:

$$\ell_k(g)\geq 0 \tag{2}$$

$$\ell_k(g_i)=\delta_{k,i} \tag{3}$$

$$\sum_{k=1}^{K_1}\ell_k(g)=1. \tag{4}$$

The model in (1) is a linear combination of stable and passive univariate models by means of a class of positive interpolation kernels [9]. Stability is automatically preserved in (1), as it is a weighted sum of stable rational macromodels. The proof of the passivity preserving property of the proposed technique over the entire design space is given in Section II-D.

C. N-D Macromodeling

The bivariate formulation can easily be generalized to the multivariate case by using multivariate interpolation methods. Multivariate interpolation can be realized in different forms: e.g., by means of tensor product [10] or by applying algorithms for scattered data such as the well-known Shepard's method [11].

1) Tensor Product Multivariate Interpolation: This technique requires that the data samples have to be located on a fully filled, but not necessarily equidistant, rectangular grid. The multivariate model can be written as

$$\mathbf{R}\left(s, g^{(1)}, \ldots, g^{(N)}\right) = \sum_{k_1=1}^{K_1} \cdots \sum_{k_N=1}^{K_N} \mathbf{R}\left(s, g_{k_1}^{(1)}, \ldots, g_{k_N}^{(N)}\right) \times \ell_{k_1}\left(g^{(1)}\right) \cdots \ell_{k_N}\left(g^{(N)}\right) \quad (5)$$

where $\ell_{k_i}(g^{(i)})$, $i = 1, \ldots, N$ satisfy all constraints (2)–(4). A suitable choice is to select each set $\ell_{k_i}(g^{(i)})$ as in piecewise linear interpolation

$$\frac{g^{(i)} - g_{k_i-1}^{(i)}}{g_{k_i}^{(i)} - g_{k_i-1}^{(i)}}, g^{(i)} \in \left[g_{k_i-1}^{(i)}, g_{k_i}^{(i)}\right], k_i = 2, \ldots, K_i \quad (6a)$$

$$\frac{g_{k_i+1}^{(i)} - g^{(i)}}{g_{k_i+1}^{(i)} - g_{k_i}^{(i)}}, g^{(i)} \in \left[g_{k_i}^{(i)}, g_{k_i+1}^{(i)}\right], k_i = 1, \ldots, K_i - 1 \quad (6b)$$

$$0, \quad \text{otherwise} \quad (6c)$$

that yields to an interpolation scheme in (5) called piecewise multilinear. This method can be also seen as a recursive implementation of simple piecewise linear interpolation.

2) Shepard's Multivariate Interpolation: Shepard's method is a standard algorithm for interpolation at nodes which have no exploitable pattern, referred to as scattered or irregularly distributed data. The interpolation kernels of Shepard's scheme also satisfy constraints (2)–(4) [9]. Unfortunately this method may present a generally unsatisfactory internodal behavior due to the occurrence of flat spots or the non-differentiability of the kernels [12].

In this letter, we use the piecewise multilinear method based on a fully filled data grid, that corresponds to the most practical way how multivariate data samples are organized and computed by a numerical simulation tool. A fully filled data grid suffers from the curse of dimensionality, but we remark that the actual interpolation process is local, because the multivariate model $\mathbf{R}(s, \vec{g})$ in a certain point $\vec{g} = (g^{(1)}, \ldots, g^{(N)})$ only depends on the *root macromodels* at the vertices of the hypercube that contains the point $\vec{g}$. An hypercube in $\mathbb{R}^N$ has 2^N vertices, 2^N increases exponentially with the number of dimensions, but it remains much smaller than the number of data samples $K_1 \cdot K_2 \cdot \ldots \cdot K_N$ in the fully filled grid. The proposed scheme avoids the unsatisfactory internodal behavior which may be present in Shepard's method. It is easy to implement and gives accurate results. We note that the kernel functions we propose only depend on the data grid points and their computation does not require the solution of a linear system to impose an interpolation constraint. The proposed parametric macromodeling technique is general and any interpolation scheme that leads to a parametric macromodel composed of a weighted sum of *root macromodels* with weights satisfying (2)–(4) can be used.

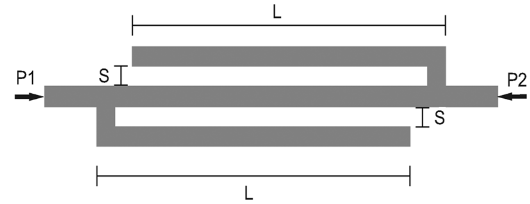

Fig. 1. Geometry of the double folded stub microstrip bandstop filter [15].

D. Passivity Preserving Interpolation

A linear network described by scattering matrix $\mathbf{S}(s)$ is passive if [13]:

1) $\mathbf{S}(s^*) = \mathbf{S}^*(s)$ for all s, where "$*$" is the complex conjugate operator.
2) $\mathbf{S}(s)$ is analytic in $\Re e(s) > 0$.
3) $\mathbf{I} - \mathbf{S}^t(s^*)\mathbf{S}(s) \geq 0; \forall s : \Re e(s) > 0$.

Concerning the *root macromodels*, conditions 1) and 2) are always satisfied since all complex poles/residues are always considered along with their conjugates and strict stability is imposed by pole-flipping. Condition 1) is preserved in (1) and the proposed multivariate extensions, as they are weighted sums with real nonnegative weights of systems respecting this first condition. Condition 2) is preserved in (1) and the proposed multivariate extensions, as they are weighted sums of strictly stable rational macromodels. Condition 3) is enforced, if needed, on the *root macromodels* by using a standard passivity enforcement technique [6]–[8]. It is equivalent to the condition $\|\mathbf{R}(s)\|_\infty \leq 1$ ($\mathbf{H}_\infty$ norm) [14], i.e., the largest singular value of $\mathbf{R}(s)$ does not exceed one in the right-half plane. Using this equivalent condition, in the bivariate case we can write:

$$\|\mathbf{R}(s, g)\|_\infty \leq \sum_{k=1}^{K_1} \|\mathbf{R}(s, g_k)\|_\infty \ell_k(g) \leq \sum_{k=1}^{K_1} \ell_k(g) = 1. \quad (7)$$

Similar results are obtained for the proposed multivariate cases, so condition 3) is satisfied by construction. We have demonstrated that all three passivity conditions for scattering representations are preserved in the novel parametric macromodeling algorithm, using the sufficient conditions (2)–(4) related to the interpolation kernels.

III. Numerical Examples

A. Double Folded Stub Microstrip Bandstop Filter

The double folded stub microstrip bandstop filter [15] under study is shown in Fig. 1. The substrate is 0.1270 mm thick with a relative dielectric constant $\epsilon_r = 9.9$ and a loss tangent $tan\delta = 0.003$. The parametric macromodel of the scattering matrix is built as function of the varying length of each folded segment $L \in [2.08 - 2.28]$ mm and varying spacing between a folded stub and the main line $S \in [0.091 - 0.171]$ mm over the frequency range [5–20] GHz. All data is simulated by ADS-Momentum[1] over a reference grid of $300 \times 60 \times 60$ samples $(freq, L, S)$.

[1]*Momentum EEsof EDA*, Agilent Technologies, Santa Rosa, CA.

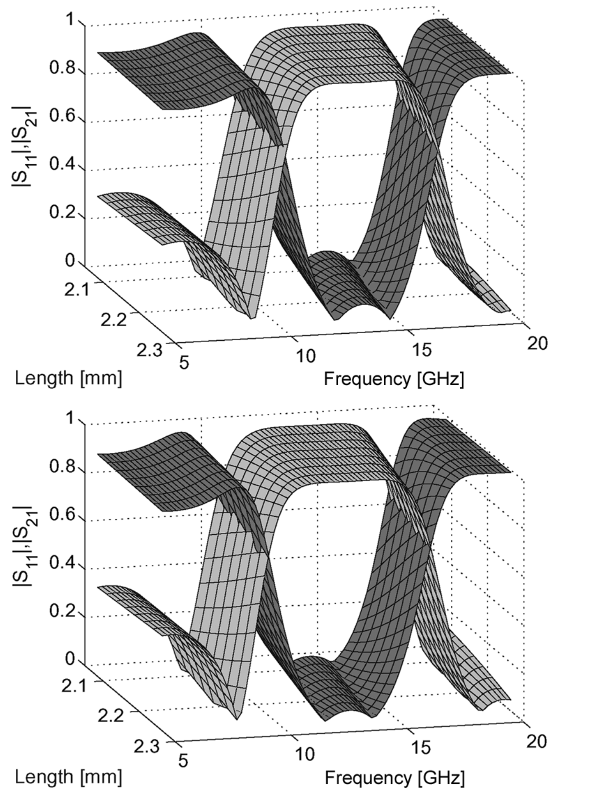

Fig. 2. Magnitude of the trivariate models of S_{11} (light grey surface) and S_{21} (dark grey surface) for $S = 0.091$ mm (top) and $S = 0.171$ mm (bottom).

441 *root macromodels* are built for 21 values of the length (L) and 21 values of the spacing (S) by means of VF. The required number of poles in VF is adaptively selected for each *root macromodel* using a bottom-up approach, in such a way that the corresponding maximum absolute model error for each entry of the scattering matrix is smaller than -60 dB. The passivity of each model is verified by checking the eigenvalues of the Hamiltonian matrix [14] and it is enforced if needed. A trivariate macromodel is obtained by piecewise multilinear interpolation of all 441 *root macromodels*. The passivity test on a dense sweep over the design space has confirmed the theoretical claim of overall passivity. Fig. 2 shows the magnitude of the parametric macromodels of $\mathbf{S}_{11}(s, L, S)$ and $\mathbf{S}_{21}(s, L, S)$ for the spacing values $S = \{0.091, 0.171\}$ mm. Fig. 3 shows the distribution of the absolute error over the dense reference grid in a histogram. The maximum absolute error over the reference grid is bounded by -64.4 dB. The parametric macromodels describe the behavior of the system very accurately, while guaranteeing overall stability and passivity.

IV. Conclusion

We have presented a new macromodeling technique for parameterized scattering representations. An efficient and reliable combination of rational identification and interpolation schemes based on a class of positive interpolation operators guarantees the overall stability and passivity of the parametric macromodel. A numerical example illustrates the capability

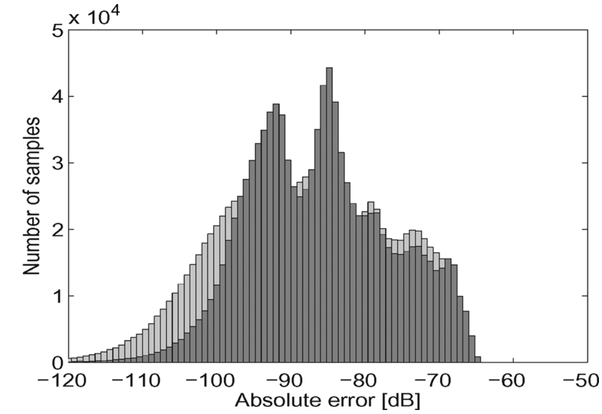

Fig. 3. Histogram: error distributions of the trivariate models of S_{11} (light grey) and S_{21} (dark grey) over 1080000 validation samples.

of the algorithm to model dynamic parameterized scattering responses very accurately, while guaranteeing stability and passivity over the complete design space.

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