Stable Parametric Macromodeling Using a Recursive Implementation of the Vector Fitting Algorithm

Tom Dhaene, *Senior Member, IEEE*, and Dirk Deschrijver

***Abstract*—A novel least-squares fitting technique is presented for the macromodeling of parameterized frequency responses. Such parametric macromodel can be used for the design, study, and optimization of microwave structures. A key benefit of the proposed method, is that the poles of the macromodel are guaranteed stable by construction. This can easily be enforced when using the presented macromodel representation.**

***Index Terms*—Parametric macromodeling, system identification, vector fitting, numerical techniques.**

I. Introduction

ROBUST parametric macromodeling is becoming increasingly important for the design, study and optimization of microwave structures. Parametric macromodels approximate the variation of the frequency domain electro-magnetic (EM) behavior of a multiport system in terms of several design variables that describe physical properties of the structure. Such macromodels are frequently used for efficient design space exploration, design optimization and sensitivity analysis.

The calculation of parametric macromodels from frequency domain responses by Vector Fitting [1] has become a topic of intense research. A robust multivariate formulation of the Orthonormal Vector Fitting technique [2] was recently introduced in [3]. It was shown that this method accurately models parameterized frequency responses with a highly dynamic behavior. However, a known drawback of the approach is that multivariate transfer function representation does not guarantee an overall stability of the poles. Some analytical criteria to detect unstable poles are provided in [4], but the enforcement of stability remains open for further research.

A solution to the stability problem is considered in [5], where a parametric macromodel is computed from time domain or frequency domain responses by barycentric interpolation of univariate nodes. It is shown that stability of the macromodel is guaranteed by construction and a simple algorithm for passivity enforcement is provided. Nevertheless, the technique poses some restrictions on the organization of the data samples, and its applicability is limited to responses that are not contaminated by detectable amounts of noise.

This letter presents a new least-squares approach to compute accurate and stable parametric macromodels from simulated frequency domain responses. It exploits the flexibility of least-squares fitting while ensuring the stability of the macromodel poles. The technique is illustrated by a parameterized lossless exponential tapered transmission line model.

Manuscript received June 01, 2008; revised November 11, 2008. First published February 11, 2009; current version published February 11, 2009. This work was supported by a research grant of the Fund for Scientific Research in Flanders (FWO Vlaanderen).

T. Dhaene is with the Department of Information Technology, Ghent University-IBBT, Ghent 9000, Belgium (e-mail: tom.dhaene@intec.ugent.be).

D. Deschrijver is with the Department of Information Technology, Ghent University-IBBT Ghent, 9000, Belgium and is also a Postdoctoral Research Fellow with the Fund for Scientific Research Flanders (FWO Vlaanderen), Brussels 1000, Belgium (e-mail:dirk.deschrijver@intec.ugent.be).

Digital Object Identifier 10.1109/LMWC.2008.2011304

II. Goal Statement

The goal of the identification algorithm is to compute a stable multivariate rational function $R(s, \boldsymbol{g})$ from simulated frequency response data $\{s, \boldsymbol{g}, H(s, \boldsymbol{g})\}$ in a least-squares sense. These data samples are usually S-parameters which depend on a complex frequency $s = j\omega$, and several real design parameters $\boldsymbol{g} = \{g^{(n)}\}_{n=1}^{N}$. The parameters $\boldsymbol{g}$ can be layout variables that describe the metallizations in an EM-circuit (such as lengths, widths, ...) or the substrate parameters (like thickness, dielectric constant, losses, ...).

III. Macromodel Identification

The multivariate rational model $R(s, \boldsymbol{g})$ is represented as a partial fraction expansion with parameterized residues [6]

$$R\left(s, g^{(1)}, \ldots, g^{(N)}\right) \approx \sum_{p=1}^{P} \frac{c_p^{(N)}\left(g^{(1)}, \ldots, g^{(N)}\right)}{s - a_p}. \quad (1)$$

The poles $\{a_p\}_{p=1}^{P}$ of the model are found by fitting the frequency responses for all geometrical parameter combinations using a common pole set. A fast QR-based implementation of the relaxed Vector Fitting technique [7] can be applied to compute them in a significantly reduced amount of time [8].

The remaining unknowns of the transfer function (1) are then the parameterized residues $c_p^{(N)}(g^{(1)}, \ldots, g^{(N)})$. If these residues are also approximated by a partial fraction expansion with common poles, then the dimension of the approximation problem decreases by 1 in each recursion step $(i = N, \ldots, 1)$

$$c_p^{(i)}\left(g^{(1)}, \ldots, g^{(i)}\right) \approx \sum_{p=1}^{P_i} \frac{c_p^{(i-1)}\left(g^{(1)}, \ldots, g^{(i-1)}\right)}{jg^{(i)} - b_p^{(i)}}. \quad (2)$$

The base case of the algorithm $(i = 1)$ reduces to a univariate macromodel identification problem which can easily be solved using standard fitting techniques, see [1] and [7] for details. It is noted that only the poles $\{a_p\}_{p=1}^{P}$ in (1) must have negative real parts to ensure overall stability of the parametric model. This can easily be enforced by flipping unstable poles into the left

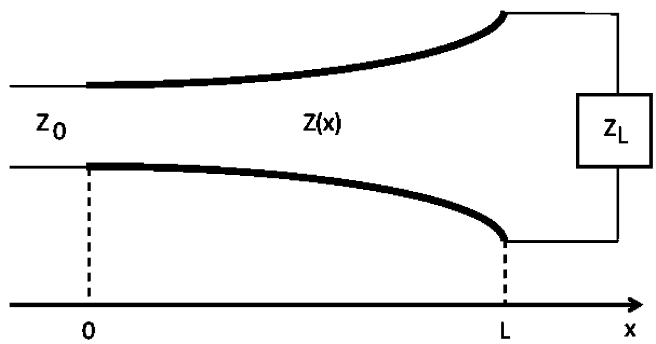

Fig. 1. Exponential tapered microstrip transmission line [10].

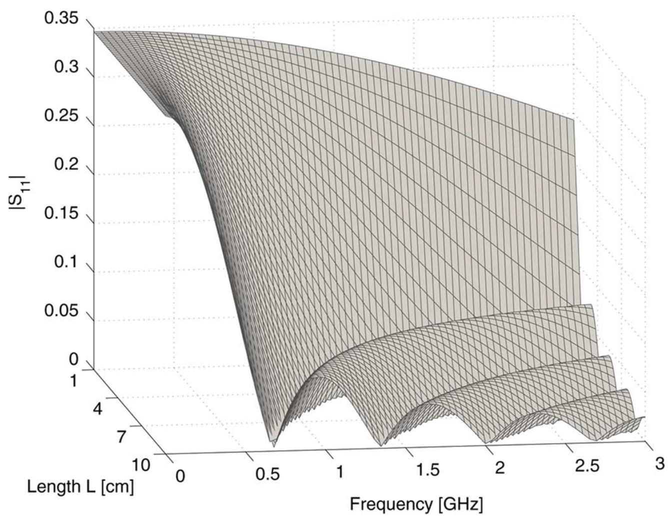

Fig. 2. Reflection coefficient S_{11} for $\varepsilon_r = 5$.

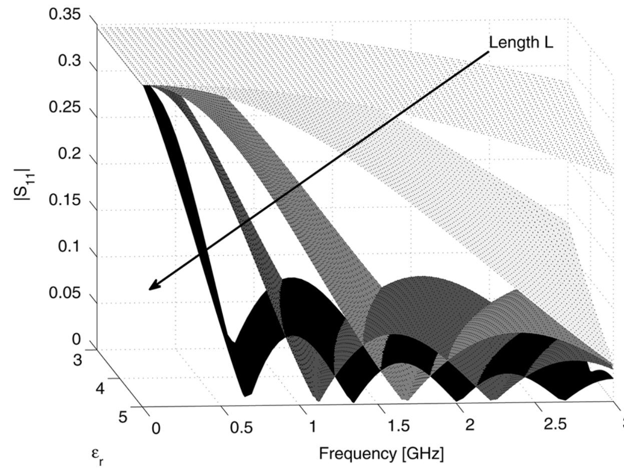

Fig. 3. Reflection coefficient S_{11} for $L = 1, 2, 4, 6, 10$ cm.

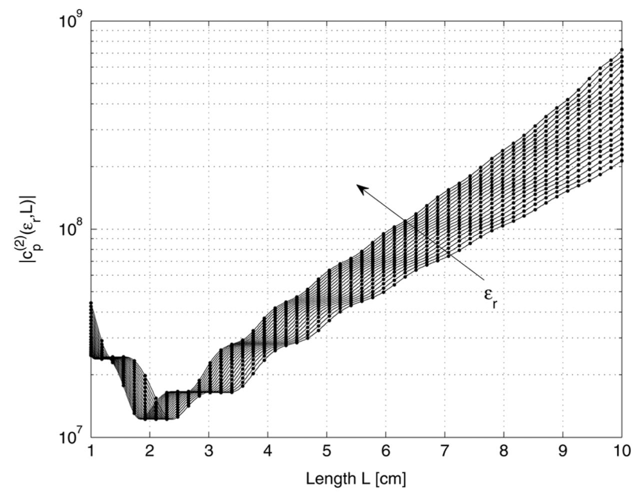

Fig. 4. Trajectories of one 2-D complex residue $c_p^{(2)}(\varepsilon_r, L)$ corresponding to pole a_p : data evaluated in 20 values of ε_r and 50 values of L (dots), fit (solid line).

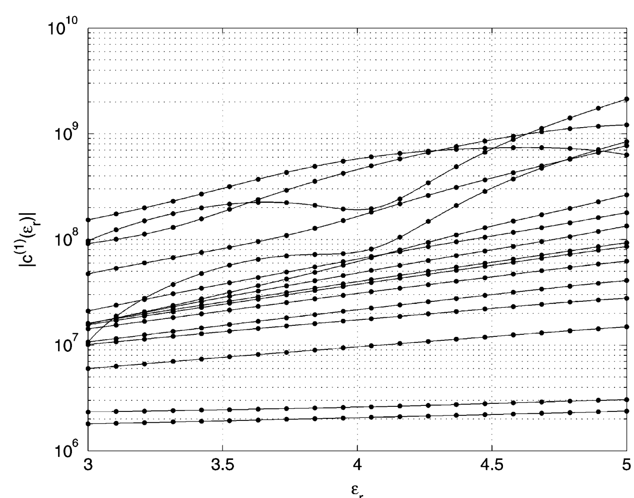

Fig. 5. Trajectories of all 1-D complex residues $c^{(1)}(\varepsilon_r)$ corresponding to all poles $b^{(2)}$ of one 2-D complex residue $c_p^{(2)}(\varepsilon_r, L)$: data evaluated in 20 values of ε_r (dots), fit (solid line).

half plane as shown in [1]. The stability of the poles $\{b_p^{(i)}\}_{p=1}^{P_i}$ in (2) is not necessary, since they do not affect the stability of the frequency-dependent macromodel in (1).

IV. 3-D Example: Transmission Line

The presented technique is used to model the reflection coefficient S_{11} of a lossless exponential tapered transmission line [9], [10] that is terminated with a matched load, as shown in Fig. 1, where $Z_0 = 50\ \Omega$ and $Z_L = 100\ \Omega$ represent the reference impedance and the load impedance, respectively.

A multivariate macromodel is computed as a function of the varying relative dielectric constant $\varepsilon_r = g^{(1)} \in [3-5]$ and varying line length $L = g^{(2)} \in [1\,\mathrm{cm} - 10\,\mathrm{cm}]$ over the frequency range [1 kHz–3 GHz]. Fig. 2 shows the frequency response of the trivariate structure for a fixed value of $\varepsilon_r = 5$, while Fig. 3 shows the variation of the response for an increasing line length L. The initial data is computed over a grid of $20(\varepsilon_r) \times 50(L) \times 50(f)$ samples, and a common set of 14 stable poles $\{a_p\}_{p=1}^{P}$ is computed, based on all 1000 univariate frequency responses. The parameterized residues $c_p^{(2)}(\varepsilon_r, L)$ corresponding to each pole a_p are then fitted as a function of the line length L for a varying relative dielectric constant ε_r using 32 common poles. A similar procedure is also applied to compute an 18-pole macromodel for the parameterized residues $c_p^{(1)}(\varepsilon_r)$ which depend solely on ε_r. As an example, some trajectories of these parameterized residues are illustrated in Figs. 4 and 5 respectively. It is noted that the frequency response should be sampled sufficiently dense, in such way that

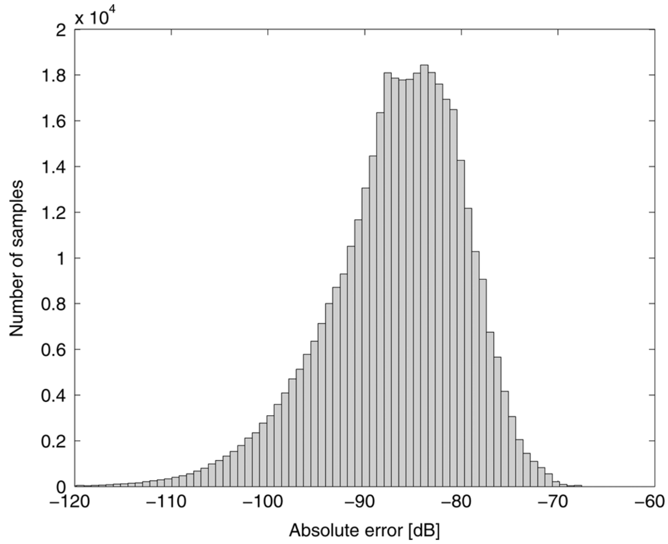

Fig. 6. Histogram: error distribution over 400000 validation samples.

the trajectories are well resolved. The total amount of model coefficients that need to be stored is 8128.

The overall macromodel is evaluated and compared over a dense set of $40 \times 100 \times 100$ validation samples, and the distribution of the absolute error is shown by a histogram in Fig. 6. It is confirmed that an overall good approximation is obtained, as the maximum error is bounded by -67.56 dB.

V. Conclusion

A new least-squares fitting approach is presented for the macromodeling of parameterized frequency responses. It reduces the sensitivity to noise on the data, that is inherent to interpolation-based approaches. The choice of the macromodel representation makes it easy to enforce stability of the poles by construction. The accuracy and robustness of the proposed method is illustrated by a parameterized lossless exponential tapered transmission line example. Although the passivity of the parametric macromodel is not enforced in this framework, the topic will be investigated in forthcoming research.

References

[1] B. Gustavsen and A. Semlyen, "Rational approximation of frequency domain responses by vector fitting," *IEEE Trans. Power Delivery*, vol. 14, no. 3, pp. 1052–1061, Jul. 1999.

[2] D. Deschrijver, B. Haegeman, and T. Dhaene, "Orthonormal vector fitting: A robust macromodeling tool for rational approximation of frequency domain responses," *IEEE Trans. Adv. Packag.*, vol. 30, no. 2, pp. 216–225, May 2007.

[3] D. Deschrijver, T. Dhaene, and D. De Zutter, "Robust parametric macromodeling using multivariate orthonormal vector fitting," *IEEE Trans. Microw. Theory Tech.*, vol. 56, no. 7, pp. 1661–1667, Jul. 2008.

[4] P. Triverio, S. Grivet-Talocia, and M. Nahkla, "An improved fitting algorithm for parametric macromodeling from tabulated data," in *Proc. IEEE Conf. Signal Propagat. Interconn.*, May 2008, p. 4.

[5] D. Deschrijver and T. Dhaene, "Stability and passivity enforcement of parametric macromodels in time and frequency domain," *IEEE Trans. Microw. Theory Tech.*, vol. 56, no. 11, pp. 2435–2441, Nov. 2008.

[6] S. Grivet-Talocia, "A parameterization scheme for lossy transmission line macromodels with application to high speed interconnects in mobile devices," *IEEE Trans. Electromagn. Compat.*, vol. 49, no. 1, pp. 18–24, Feb. 2007.

[7] B. Gustavsen, "Improving the pole relocating properties of vector fitting," *IEEE Trans. Power Delivery*, vol. 21, no. 3, pp. 1587–1592, Jul. 2006.

[8] D. Deschrijver, M. Mrozowski, T. Dhaene, and D. De Zutter, "Macromodeling of multiport systems using a fast implementation of the vector fitting technique," *IEEE Microw. Wireless Compon. Lett.*, vol. 18, no. 6, pp. 383–385, Jun. 2008.

[9] D. M. Pozar, *Microwave Engineering*, 2nd ed. New York: Wiley, 1998.

[10] M. Hadi Eghlidi, K. Mehrany, and B. Rashidian, "Analytical approach for analysis of nonuniform lossy/lossless transmission lines and tapered microstrips," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 12, pp. 4122–4129, Dec. 2006.