

# Efficient Time-Domain Macromodeling of Complex Interconnection Structures

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**Abstract**—The Vector Fitting algorithm [1] is an iterative procedure to compute rational approximations of frequency-domain responses. It was shown that the robustness of this technique can be enhanced by using a set of orthonormal rational basis functions, leading to the Orthonormal Vector Fitting method [2]. In this paper, a time-domain implementation of this method is proposed for the macromodeling of transient port responses. It is shown that this method is more robust towards the initial pole specification, when compared to the classical time-domain Vector Fitting method [3].

**Keywords**—Time domain macromodeling, System identification, Orthonormal Vector Fitting.

## I. INTRODUCTION

Compact and accurate macromodels are crucial for accurate system-level simulations. The derivation of such models from measurements or first-principle simulators is numerically not a trivial task, even for linear systems. In the frequency-domain, the Vector Fitting (VF) method [1] was proposed to calculate broadband transfer functions from a given frequency-domain response. It was shown in [3] that this method can be extended to model transient port responses in the time-domain.

Initially, both methods start from a prescribed set of poles which define the basis functions of the approximation problem. Using an iterative least-squares method, these poles are relocated in a two-step procedure to minimize the global fitting error.

It was shown in [2] that the numerical robustness of the technique can be improved by applying a Gram-Schmidt orthonormalization procedure on the basis functions, leading to the Orthonormal Vector Fitting (OVF) method. In this paper, an analogous reasoning is applied to build macromodels in the time-domain. This way, the conditioning of the system equations becomes less sensitive to the initial pole specification, and accurate models can be computed in fewer iterations. This improves the efficiency of the method and reduces the overall computation time.

## II. MODEL REPRESENTATION

Frequency-domain macromodelling tools are used to build a rational transfer function  $R(s)$ , based on the

spectral response  $(s, H(s))$  of a physical structure.

$$R(s) = \frac{N(s)}{D(s)} = \frac{\sum_{p=1}^P c_p \Phi_p(s, a)}{\tilde{c}_0 + \sum_{p=1}^P \tilde{c}_p \Phi_p(s, a)} \quad (1)$$

In the frequency-domain OVF technique, numerator and denominator are expanded as a linear combination of orthonormal rational functions  $\Phi_p(s, a)$ , which are based on a prescribed set of stable poles  $a = \{-a_1, \dots, -a_P\}$  [2]. If  $-a_p$  is a real pole, then the basis functions are defined as

$$\Phi_p(s, a) = \frac{\sqrt{2 \Re e(a_p)}}{s + a_p} \left( \prod_{j=1}^{p-1} \frac{s - a_j^*}{s + a_j} \right) \quad (2)$$

and a linear combination is formed when two poles  $-a_p = -a_{p+1}^*$  form a complex conjugate pair

$$\Phi_p(s, a) = \frac{\sqrt{2 \Re e(a_p)}(s - |a_p|)}{(s + a_p)(s + a_{p+1})} \left( \prod_{j=1}^{p-1} \frac{s - a_j^*}{s + a_j} \right) \quad (3)$$

$$\Phi_{p+1}(s, a) = \frac{\sqrt{2 \Re e(a_p)}(s + |a_p|)}{(s + a_p)(s + a_{p+1})} \left( \prod_{j=1}^{p-1} \frac{s - a_j^*}{s + a_j} \right) \quad (4)$$

It can be shown that these basis functions are orthonormal with respect to the following inner product ( $1 \leq m, n \leq P$ )

$$\langle \Phi_m(s), \Phi_n(s) \rangle_s = \frac{1}{2\pi i} \int_{i\mathbb{R}} \Phi_m(s) \Phi_n^*(s) ds \quad (5)$$

## III. TRANSFER FUNCTION IDENTIFICATION

The goal of the identification process, is to identify the coefficients  $c_p$  and  $\tilde{c}_p$  in (1), such that the difference between  $R(s)$  and  $H(s)$  is minimized in a least-squares sense. A linear approximation of this non-linear identification problem is obtained by solving the following set of equations ( $\tilde{c}_0 = 1$ )

$$\sum_{p=1}^P c_p \Phi_p(s, a) - H(s) \sum_{p=1}^P \tilde{c}_p \Phi_p(s, a) \approx H(s). \quad (6)$$

In successive iterations, a Sanathanan-Koerner iteration [5], [6] can be applied which iteratively relocates the

transfer function poles, until a good approximation is obtained [7]. It is found that an accurate fitting model usually results in a few iterations, provided that the initial set of starting poles is well-chosen.

To obtain the time-domain algorithm, equation (6) can be rewritten in terms of an input signal  $u(t)$  and the corresponding output signal  $y(t)$ . To this end, the result is transformed to the time-domain by multiplying equation (6) by  $U(s)$ , which represents the Laplace transform of  $u(t)$ .

If  $\phi_p(t, a)$  denotes the Inverse Laplace Transform of  $\Phi_p(s, a)$ , and  $f(t) \star g(t)$  the convolution of  $f(t)$  and  $g(t)$ , then it follows that

$$\sum_{p=1}^P c_p (u(t) \star \phi_p(t, a)) - \sum_{p=1}^P \tilde{c}_p (y(t) \star \phi_p(t, a)) \approx y(t) \quad (7)$$

Since orthogonality in the frequency domain corresponds to orthogonality in the time-domain, it is clear that the relevant time-domain basis functions  $\phi_p(t)$  are given by the inverse Laplace transform of  $\Phi_p(s)$ , and that they are orthonormal with respect to the time-domain inner product ( $1 \leq m, n \leq P$ )

$$\langle \phi_m(t), \phi_n(t) \rangle_t = \int_0^\infty \phi_m(t) \phi_n(t) dt \quad (8)$$

The fact that no explicit expression is available for these basis functions is of no importance, because equation (7) only needs the convolution of these functions with the input and output signals  $u(t)$  and  $y(t)$ . To compute the filtered signals  $u(t) \star \phi_p(t)$  or  $y(t) \star \phi_p(t)$ , the state space realization of the orthonormal basis functions  $\Phi_p$  can be simulated with input  $u(t)$  or  $y(t)$ , respectively. More details about the construction of this realization are described in [4].

A direct application of equation (7) to the time-domain samples  $(t_k, u(t_k), y(t_k))$  leads to a system of equations which are linear in terms of the coefficients  $c_p$  and  $\tilde{c}_p$ . Based on the values of the coefficients  $\tilde{c}_p$ , the poles of the transfer function can be calculated by solving an eigenvalue problem [4]. The iterative replacement of the prescribed poles by the relocated poles is repeated until convergence is detected.

In the final iteration, the converged poles  $\theta = \{-\theta_1, \dots, -\theta_P\}$  can be used to identify the coefficients  $\gamma_p$  of the convoluted exponential terms as a linear problem

$$\sum_{p=1}^P \gamma_p (u(t) \star e^{-\theta_p t}) = y(t) \quad (9)$$

#### IV. NUMERICAL EXAMPLE

A passive system is excited with a Gaussian pulse, which is centered at  $t = 0.6$  ns, with a width of 0.2 ns and a height of 1 in normalized units. The transient response  $y(t)$  is approximated by a 104-pole transfer function using the VF and OVF algorithm.

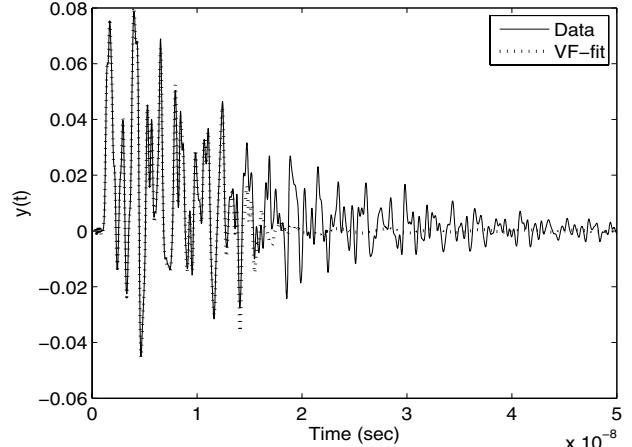


Fig. 1. VF approximation of transient response (1 iteration,  $v = 0.1$ )

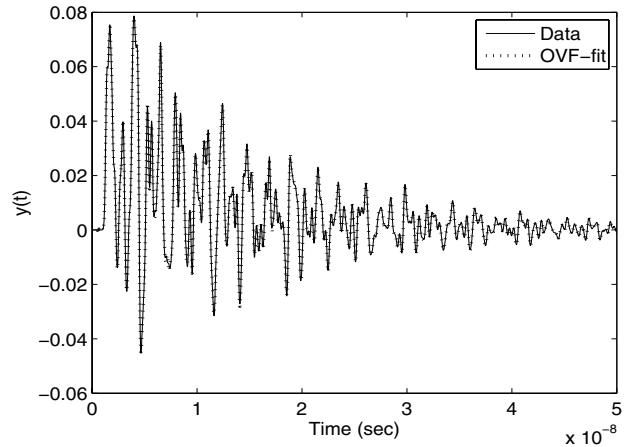


Fig. 2. OVF approximation of transient response (1 iteration,  $v = 0.1$ )

The initial set of 104 starting poles is typically chosen according to the heuristical scheme proposed in [1]

$$\begin{aligned} -a_p &= -\alpha + \beta i, -a_{p+1} = -\alpha - \beta i \\ \alpha &= v\beta \end{aligned} \quad (10)$$

provided that the imaginary parts  $\beta$  are linearly distributed over the frequency range [0 – 3 GHz], and  $v = 0.01$ . Chosing small values for  $v$  often results in a well-conditioned system of equations, which can be solved accurately by both techniques. However, it is shown in [1] that the VF approach suffers numerical ill-conditioning if larger values for  $v$  are selected.

To illustrate the improvement of the OVF technique, the value of  $v$  is increased to 0.1, and both algorithms are applied to model the time-domain data in a single iteration. As can be seen from Figures 1 and 2, the VF model only captures the initial transient response, while the OVF model has a higher overall accuracy.

Table 1 illustrates the evolution of the maximal absolute fitting error if multiple iterations are performed. As can be seen, the VF method converges to comparable results

TABLE I  
MAXIMAL ABSOLUTE TIME-DOMAIN ERROR ( $v = 0.1$ )

Max. Abs. Error	VF	OVF
Iteration 1	0.0307	0.0024
Iteration 2	0.0031	0.0021
Iteration 3	0.0019	0.0025
Iteration 4	0.0021	0.0011
Iteration 5	0.0011	0.0004
Iteration 6	0.0005	0.0004

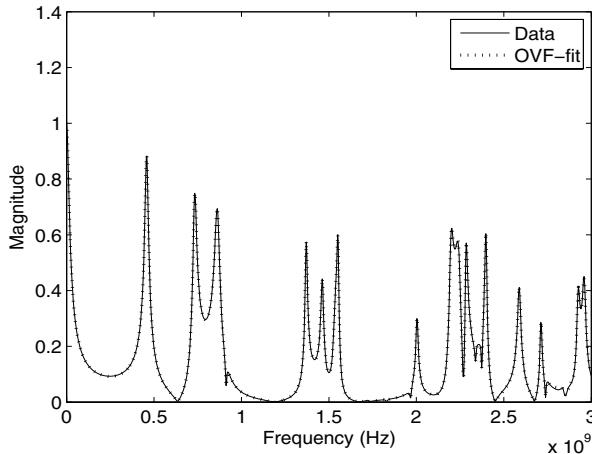


Fig. 3. Frequency-domain simulation of OVF model

if sufficient iterations are performed. However, it should be noted that additional iterations lead to an undesired increase in computation time. Therefore, the OVF method is preferable due to its robustness and efficiency.

As a means of validation, the OVF model in the final iteration is simulated in the frequency-domain over the frequency range of interest, and compared to the reference response. As can be seen from Figure 3, an overall good approximation is obtained.

## V. CONCLUSIONS

In this paper, the use of Orthonormal Vector Fitting for time-domain identification of transient port responses is proposed. It is shown that the method is less sensitive to the initial pole specification, as compared to the classical time-domain Vector Fitting technique [3]. Since less iterations are needed to obtain accurate results, the overall computation time can be reduced.

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