

Generalised vector fitting algorithm for macromodelling of passive electronic components

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A popular rational broadband macromodelling algorithm, called vector fitting, is generalised to also include frequency derivatives of the spectral data samples. The use of higher-order frequency derivatives can significantly reduce the overall computational cost of the simulation and modelling process.

Introduction: Compact rational macromodels, based on measurements or full-wave EM simulations, are very important for efficient time domain and frequency domain simulation of high-speed interconnection structures.

In [1], the vector fitting (VF) technique was introduced for efficient frequency domain identification of passive components, based on a set of frequency samples. In this Letter, the basic VF technique is generalised, to include higher-order frequency derivatives in the fitting process. Frequency derivatives are scaled moments (i.e. coefficients of a Taylor series around a given expansion point) [2], and can often be simulated at a significantly lower computational cost than additional frequency samples.

Rational fitting technique: vector fitting: The frequency domain behaviour of linear time-invariant (LTI) systems can be modelled by a rational pole-residue model

$$R(s_k) = \sum_{p=1}^P \frac{c_p}{s_k - a_p} + d + s_k h \quad (1)$$

$R(s_k)$ approximates the simulated data samples $H(s_k)$ and their t th order frequency derivatives $H^{(t)}(s_k)$, relative to s , at the discrete complex frequencies $s_k, \forall k=0, \dots, K$, and $\forall t=0, \dots, T$. a_p and c_p are the poles and residues, respectively, $\forall p=1, \dots, P$. d is a constant and h is an optional linear factor. The vector fitting (VF) technique [1] linearises the nonlinear identification problem, similar to the Steiglitz-McBride iteration [3]. VF starts with an initial guess of P poles, and converges towards a global broadband model by relocating the poles in an iterative way. The unknown system variables are estimated by solving two consecutive linear least-squares fits, and it is ensured that the poles and residues are real or occur in complex conjugate pairs. Bounded-input-bounded-output (BIBO) system stability is enforced by the algorithm.

Rational fitting technique: VF + frequency derivatives: The ‘weighted’ spectral behaviour of an LTI system can be approximated by a rational function based on an initial set of starting poles \bar{a}_p . This rational function approximates (1) multiplied by an unknown rational ‘weighting’ function $\sigma(s)$. Since $\sigma(s)$ is also rational, it can be represented in pole-residue form, which leads to the following augmented problem

$$\begin{bmatrix} \sigma(s)R(s) \\ \sigma(s) \end{bmatrix} = \begin{bmatrix} \sum_{p=1}^P \frac{c_p}{s - \bar{a}_p} + d + sh \\ \sum_{p=1}^P \frac{\tilde{c}_p}{s - \bar{a}_p} + 1 \end{bmatrix} \quad (2)$$

The augmented problem can be linearised in function of the unknowns c_p, d, h and \tilde{c}_p by multiplying the second line of the vector equation with $R(s)$.

$$\left(\sum_{p=1}^P \frac{c_p}{s - \bar{a}_p} + d + sh \right) = \left(\sum_{p=1}^P \frac{\tilde{c}_p}{s - \bar{a}_p} + 1 \right) R(s) \quad (3)$$

or

$$(\sigma R)_{fit}(s) = \sigma_{fit}(s)R(s) \quad (4)$$

Equating $R(s)$ to $H(s)$ for all frequency samples, and deriving the left hand side and right hand side of (3) t times leads to the following system of equations,

$$A_k x = b_k \quad (5)$$

for each discrete complex frequency s_k , and where

$$A_k = \begin{bmatrix} \psi_{1,0}(s_k) & \dots & \psi_{P,0}(s_k) & 1 & s_j & \phi_{1,0}(s_k) & \dots & \phi_{P,0}(s_k) \\ \psi_{1,1}(s_k) & \dots & \psi_{P,1}(s_k) & 0 & 1 & \phi_{1,1}(s_k) & \dots & \phi_{P,1}(s_k) \\ \dots & \dots \\ \psi_{1,T}(s_k) & \dots & \psi_{P,T}(s_k) & 0 & 0 & \phi_{1,T}(s_k) & \dots & \phi_{P,T}(s_k) \end{bmatrix} \quad (6)$$

$$x = [c_1 \dots c_P \ d \ h \ \tilde{c}_1 \ \dots \ \tilde{c}_P]^T \quad (7)$$

$$b_k = [H^{(0)}(s_k) \ H^{(1)}(s_k) \ \dots \ H^{(T)}(s_k)]^T \quad (8)$$

$\psi_{p,t}(s_k)$ is defined as

$$\psi_{p,t}(s_j) = \frac{d^t}{ds^t} [(s_k - \bar{a}_p)^{-1}] = (-1)^t t! (s_k - \bar{a}_p)^{-(t+1)} \quad (9)$$

and based on Leibniz’ identity, $\phi_{p,t}(s_k)$ represents

$$\begin{aligned} \phi_{p,t}(s_k) &= \frac{d^t}{ds^t} [-H(s_k)(s_k - \bar{a}_p)^{-1}] \\ &= - \sum_{\tau=0}^t \binom{t}{\tau} H^{(t-\tau)}(s_k) \psi_{p,\tau}(s_k) \end{aligned} \quad (10)$$

To ensure that complex poles, and associated residues appear in complex conjugate pairs, the following transformation is introduced.

Suppose that the poles a_p and a_{p+1} constitute a complex conjugate pair $a_p = d' + ja''$, $a_{p+1} = d' - ja''$. Then the corresponding vector columns $A_{k,p}$ and $A_{k,p+1}$ of (6) can be replaced by $A'_{k,p} = \psi_{p,t}(s_k) + \psi_{p+1,t}(s_k)$ and $A''_{k,p+1} = j\psi_{p,t}(s_k) - j\psi_{p+1,t}(s_k)$, such that the residues $c_p = c' + jc''$ and $c_{p+1} = c - jc''$ satisfy the complex conjugacy requirement as well. This has the effects that the corresponding residues in the solution vector x become equal to c' and c'' , respectively. The transformation for the residues of $\sigma(s)$ is similar.

Writing (6) and (8) for all frequencies s_k and separating the real and imaginary part, leads to the following least-squares system, and ensures that all coefficients of the rational functions are real

$$\begin{bmatrix} \Re(A) \\ \Im(A) \end{bmatrix} x = \begin{bmatrix} \Re(b) \\ \Im(b) \end{bmatrix} \quad (11)$$

After the parameterisation of the rational model, the following function approximations are obtained

$$(\sigma R)_{fit}(s) = \alpha \frac{\prod_{n=1}^{P+1} (s - z_n)}{\prod_{n=1}^P (s - \bar{a}_n)} \quad \text{and} \quad \sigma_{fit}(s) = \frac{\prod_{n=1}^P (s - \tilde{z}_n)}{\prod_{n=1}^P (s - \bar{a}_n)} \quad (12)$$

From (12), $R(s)$ can be calculated

$$R(s) = \frac{(\sigma R)_{fit}(s)}{\sigma_{fit}(s)} = \alpha \frac{\prod_{n=1}^{P+1} (s - z_n)}{\prod_{p=1}^P (s - \tilde{z}_n)} \quad (13)$$

The initial poles are cancelled out, such that the new poles of $R(s)$ equal the zeros of $\sigma_{fit}(s)$. This process is called pole relocation. Unstable poles are flipped into the left half plane to ensure system stability. To calculate the residues of $R(s)$, (1) is solved with the zeros of $\sigma(s)$ as new poles for $R(s)$. This leads to a similar least-squares problem as (11) where the unknown parameters are now c_p, d and h . A detailed analysis of the significance of the starting pole locations can be found in [1].

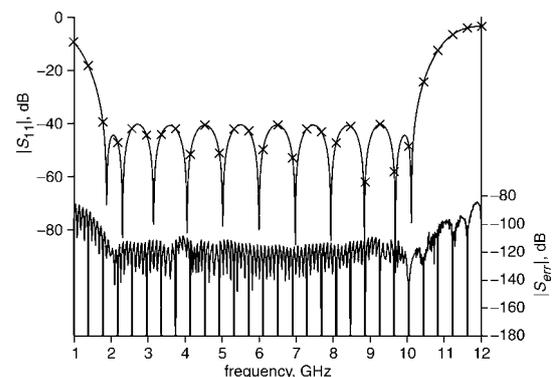


Fig. 1 Magnitude data against model and error
29 samples plus no derivatives

Example: quarter wavelength filter: The reflection coefficient S_{11} of a quarter wavelength filter is modelled over the frequency range 1–50 GHz. The desired model accuracy is set to -80 dB, or better. When no frequency derivatives are used (see Fig. 1), 29 equidistant samples are required to obtain the desired accuracy. When the first and second derivatives are included, only 15 (see Fig. 2) and 10 (see Fig. 3) equidistant samples are required. An adaptive frequency sampling (AFS) algorithm, as described in [4], could also be used.

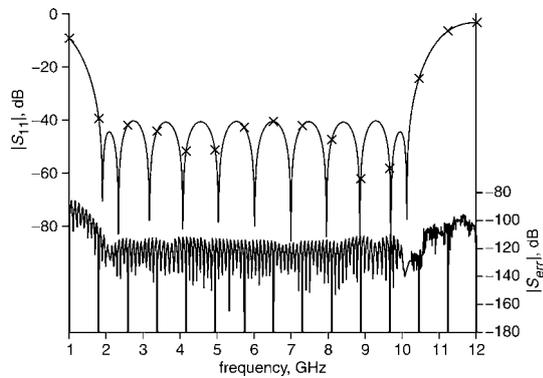


Fig. 2 Magnitude data against model and error
15 samples plus first derivatives

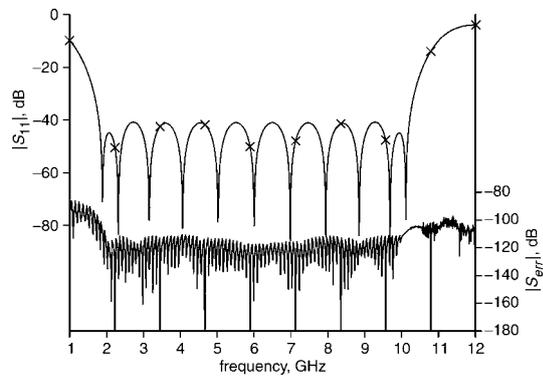


Fig. 3 Magnitude data against model and error
10 samples plus first and second derivatives

Conclusion: A popular and robust rational fitting technique, called vector fitting, was generalised to fit spectral data samples and/or frequency derivatives. Higher-order frequency derivatives can often be obtained at a relatively low additional computational cost, and provide valuable information. Including this information can reduce significantly the overall simulation time.

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References

- 1 Gustavsen, B., and Semlyen, A.: 'Rational approximation of frequency domain responses by vector fitting', *IEEE Trans. Power Deliv.*, 1999, **14**, (1), pp. 1052–1061
- 2 Chen, H., Zheng, J., and Fang, J.: 'Multipoint moment matching based model generation for complex systems', *Electrical Performance on Electronic Packaging*, 2003, pp. 299–302
- 3 Shaw, A.K.: 'An optimal method for identification of pole-zero transfer functions'. *IEEE Int. Symp. on Circuits and Systems*, 1992, Vol. 5, pp. 2409–2412
- 4 Deschrijver, D., and Dhaene, T.: 'Efficient GA-inspired macromodeling of general LTI multiport systems'. *8th IEEE Workshop on Signal Propagation on Interconnects*, 2004, pp. 95–98