

Scattering matrix representation for the incidence of electromagnetic waves on multiconductor transmission line structures.

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Abstract - In the present contribution, we demonstrate how circuits described by a coupled transmission line model with distributed sources can be represented by a classical $\underline{\underline{S}}$ -matrix description. This approach allows one to use commercially available time and/or frequency domain simulators to study the circuit behaviour.

circuits with distributed sources, one is obliged to develop a new simulator or one has to look for an alternative formulation. In the next section (1) will be transformed into a description that can be handled by modern circuit simulators .

GEOMETRY AND FORMULATION OF THE PROBLEM

In [1, 2, 3, 4] amongst others circuit models are derived to account for the influence of an incident field impinging on a multiconductor transmission line system (MTL). This MTL with constant cross-section consists of N conductors. In many papers, the conductors are assumed to be cables above earth and located in free space ([1], [2]), in other contributions, the conductors may be embedded in a planar stratified medium ([3], [4]). In both cases, the coupled transmission line model one obtains to account for the influence of the incident field has the following form (the z-direction equals the propagation direction) :

$$\begin{aligned} \frac{d\underline{\underline{V}}(z, \omega)}{dz} + \underline{\underline{Z}}(\omega)\underline{\underline{I}}(z, \omega) &= \underline{\underline{V}}_g(z, \omega) \\ \frac{d\underline{\underline{I}}(z, \omega)}{dz} + \underline{\underline{Y}}(\omega)\underline{\underline{V}}(z, \omega) &= \underline{\underline{I}}_g(z, \omega) \end{aligned} \quad (1)$$

$\underline{\underline{V}}(z, \omega)$, $\underline{\underline{I}}(z, \omega)$, $\underline{\underline{V}}_g(z, \omega)$, $\underline{\underline{I}}_g(z, \omega)$:
vectors of N elements,
 $\underline{\underline{Z}}(\omega)$ ($\underline{\underline{Y}}(\omega)$) :
NxN impedance (admittance) matrix.

$\underline{\underline{V}}_g(z, \omega)$ and $\underline{\underline{I}}_g(z, \omega)$ are frequency and z-dependent, distributed sources. As most commercially available circuit simulators do not allow transmission line

SCATTERING MATRIX DESCRIPTION OF THE PROBLEM

It is generally known that a circuit with active components can be described by an $\underline{\underline{S}}$ -matrix formalism in the following way :

$$\underline{\underline{B}}(\omega) = \underline{\underline{S}}(\omega)\underline{\underline{A}}(\omega) + \underline{\underline{C}}(\omega). \quad (2)$$

$\underline{\underline{A}}(\omega)$, $\underline{\underline{B}}(\omega)$, $\underline{\underline{C}}(\omega)$: vectors of N elements,
 $\underline{\underline{S}}(\omega)$: (NxN)-matrix.

$\underline{\underline{S}}(\omega)$ is the global scattering matrix and $\underline{\underline{C}}(\omega)$ the external source vector. $\underline{\underline{S}}(\omega)$, $\underline{\underline{A}}(\omega)$, $\underline{\underline{B}}(\omega)$ and $\underline{\underline{C}}(\omega)$ may be frequency dependent.

In the following the voltage wave vectors of the structure, described by equation (2), are defined with respect to the characteristic impedance matrix $\underline{\underline{Z}}_c$ of the structure. $\underline{\underline{A}}^1(\omega)$ and $\underline{\underline{B}}^1(\omega)$ ($\underline{\underline{A}}^2(\omega)$ and $\underline{\underline{B}}^2(\omega)$) represent the incident and reflected voltage wave vectors at the beginning (end) of the conductors (see Fig. 1). As the conductors may be coupled, the voltages and impedances in Fig. 1 at the line ends are represented by matrices. With these definitions, one obtains the following $\underline{\underline{S}}$ -matrix description for the MTL structure under study (The superscript indicates the port number, the subscript gives the dimension of the matrix ; d is the length of the considered MTL-

section and $\underline{\underline{A}}_v$ is the complex voltage propagation matrix of the MTL).

$$\begin{pmatrix} \underline{\underline{B}}^1(\omega)_{N \times 1} \\ \underline{\underline{B}}^2(\omega)_{N \times 1} \end{pmatrix} = \underline{\underline{B}}_{2N \times 1}$$

$$= \begin{pmatrix} 0_{N \times N} & \frac{1}{\underline{\underline{Z}}_c} e^{-\underline{\underline{A}}_v d} \frac{1}{\underline{\underline{Z}}_c} \\ \frac{1}{\underline{\underline{Z}}_c} e^{-\underline{\underline{A}}_v d} \frac{1}{\underline{\underline{Z}}_c} & 0_{N \times N} \end{pmatrix} \begin{pmatrix} \underline{\underline{A}}^1(\omega) \\ \underline{\underline{A}}^2(\omega) \end{pmatrix}$$

$$+ \begin{pmatrix} -\frac{1}{2} \frac{1}{\underline{\underline{Z}}_c} e^{-\frac{1}{2} \underline{\underline{A}}_v \text{begin}} \int_{\text{begin}}^{\text{end}} e^{-\underline{\underline{A}}_v z} (\underline{\underline{V}}_g - \underline{\underline{Z}}_c \underline{\underline{I}}_g) dz \\ \frac{1}{2} \frac{1}{\underline{\underline{Z}}_c} e^{-\frac{1}{2} \underline{\underline{A}}_v \text{end}} \int_{\text{begin}}^{\text{end}} e^{\underline{\underline{A}}_v z} (\underline{\underline{V}}_g + \underline{\underline{Z}}_c \underline{\underline{I}}_g) dz \end{pmatrix}$$

$$= \underline{\underline{S}}_{2N \times 2N} \underline{\underline{A}}_{2N \times 1} + \underline{\underline{C}}_{2N \times 1}$$

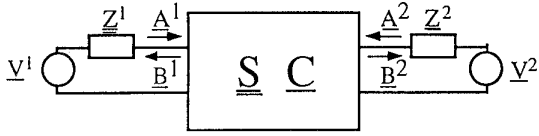


Fig. 1 : A 2N coupled port system characterised by a $\underline{\underline{S}}$ - and $\underline{\underline{C}}$ -matrix.

This means that when the influence of an incident field on a MTL with N conductors is studied the active part of the structure is represented by a $[2N \times 1]$ - $\underline{\underline{C}}$ -matrix while the passive part is described by the $[2N \times 2N]$ - $\underline{\underline{S}}$ -matrix. This system can be transformed to an equivalent network, described by an active $[4N \times 4N]$ - $\underline{\underline{S}}$ -matrix and with 2N additional external sources :

$$\begin{pmatrix} \underline{\underline{B}}_{2N \times 1} \\ \underline{\underline{C}}_{2N \times 1} \end{pmatrix} = \begin{pmatrix} \underline{\underline{S}}_{2N \times 2N} & \underline{\underline{I}}_{2N \times 2N} \\ \underline{\underline{0}}_{2N \times 2N} & \underline{\underline{I}}_{2N \times 2N} \end{pmatrix} \begin{pmatrix} \underline{\underline{A}}_{2N \times 1} \\ \underline{\underline{C}}_{2N \times 1} \end{pmatrix} \quad (4)$$

where $\underline{\underline{I}}_{2N \times 2N}$ is the $2N \times 2N$ unitary matrix.

To obtain this, 2N additional ports are added to the circuit (Fig. 2). Each of these new ports is driven by a voltage source, whose variation as a function of frequency is given by one of the elements of the $\underline{\underline{C}}$ -matrix (one port for each element) multiplied by 2 times the square root of the reference impedance of the port. The impedance at each coupled port can be chosen freely. With this transformation, the internal active part $\underline{\underline{C}}$ (i.e. the sources $\underline{\underline{V}}_g$ and $\underline{\underline{I}}_g$) are

represented as completely outside sources. This approach makes it possible to use widely available simulators such as HP-MDS and Libra to characterise the structures described by equation (1).

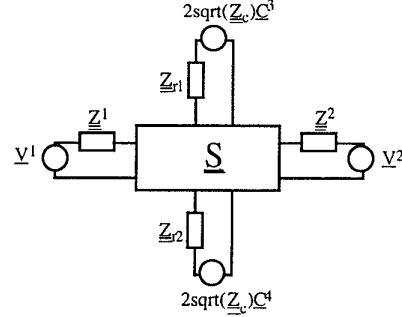


Fig. 2 : Active 4N coupled port system characterised by a $\underline{\underline{S}}$ -matrix.

NUMERICAL RESULTS

Consider the configuration shown in Fig. 3. Two asymmetric strips are placed on a lossy substrate, they are 15 mm long and are terminated as shown in Fig. 4. The voltage source generates a sinusoidal wave of 10GHz with an amplitude of 2V. This structure is illuminated by a horizontal electrical dipole $\underline{\underline{J}}_h = \underline{\underline{J}} \underline{\underline{u}}_x \delta(x) \delta(z) \delta(y-2\text{mm})$ with $J=1\text{mA/mm}$, situated at 2 mm above the ground plane.

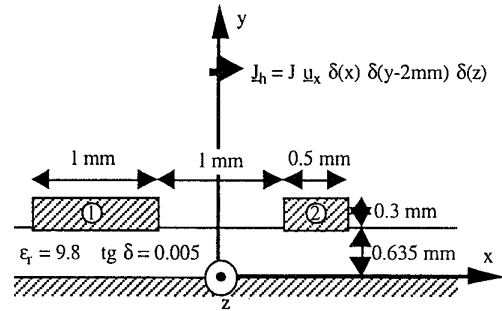


Fig. 3 : Cross-section of the structure under study.

Fig. 5 shows the induced voltage at the beginning and at the end of the left and of the right conductor respectively, in the presence of the electrical dipole. In Fig. 6 the dipole is removed. One notices that only in node 1 the incident field couples destructively, in all the other nodes, the voltage increases due to the presence of the interfering source.

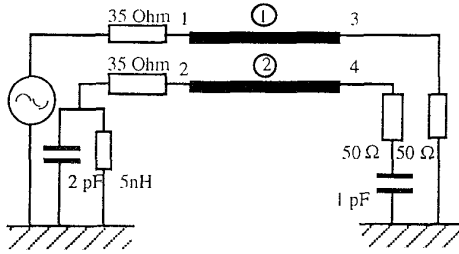


Fig. 4 : Line terminations of the structure under study.

CONCLUSION

A method to transform a circuit model with distributed voltage and current sources into a classical \underline{S} -matrix description is demonstrated. It is shown that in this way it is possible to use commonly available circuit simulators to study the circuit behaviour.

REFERENCES

- [1] C. R. Paul, "Frequency response of multiconductor transmission lines illuminated by an electromagnetic field,"

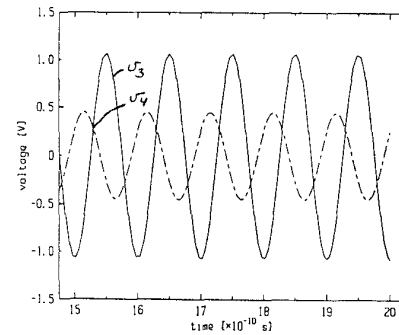
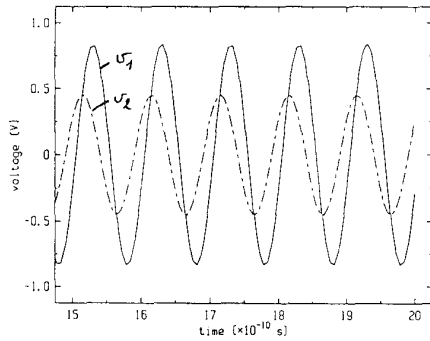


Fig. 5 : Voltage at the line ends, in the presence of the dipole.

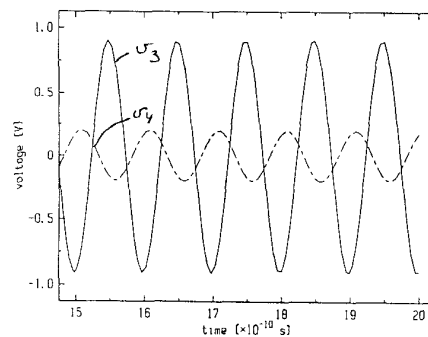
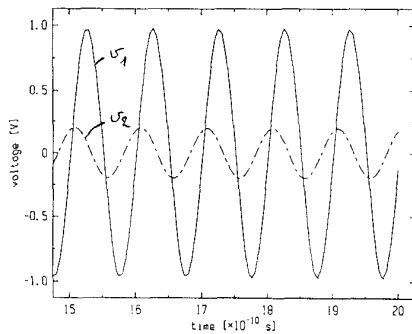


Fig. 6 : Voltage at the line ends, in the absence of the dipole.

IEEE Trans Electromag. Compat., vol. EMC-18, no. 4, pp 183-190, Nov. 1976.

- [2] M. Ianoz, C. A. Nucci and F. M. Tesche, "Transmission line theory for field to transmission lin coupling calculations," *Electromagnetics*, vol. 8, pp. 171-211, 1988.
- [3] D. De Zutter and F. Olyslager, "High-frequency circuit model for the incidence of electromagnetic waves on hybrid waveguide structures," *Proceedings 24th European Microwave Conference 1994*, pp 279-283, Sep. 1994.
- [4] I. Wuys and D. De Zutter, "Circuit model for plane wave incidence on multiconductor Transmission lines," *IEEE Trans Electromag. Compat.*, vol. EMC-36, no. 3, Aug. 1994.

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