Transactions Briefs

Accurate Transient Simulation Algorithm for High-Speed Interconnections

T. Dhaene, L. Martens, and D. De Zutter

***Abstract*—In this letter, we present a new accurate transient simulation algorithm for high-speed interconnection structures consisting of general uniform dispersive and lossy multiconductor transmission lines terminated in arbitrary loads. All ports of the interconnection structure are modeled as extended Thevenin models.**

I. Introduction

The study of transient propagation phenomena on general interconnection structures is very important for the synthesis and the design of high-speed electronic circuits. Transient analysis of general uniform transmission line structures has been extensively studied in the past [1]–[2]. In this letter, the transient simulation of general uniform dispersive lossy multiconductor transmission lines is handled in a time-stepping way. At each point in time, all ports of the high-speed interconnection structure are modeled as equivalent Thevenin networks. The new simulation method which is presented here is based on a simulation algorithm for nonuniform structures characterized by S-parameters [3]. The frequency dependence of the coupling between different ports of the interconnection structure is explicitly taken into account. The advantage of this method over the other techniques resides in its flexibility, its computational stability, its noniterative character, its simple Thevenin representation and its compatibility with existing circuit simulators such as SPICE.

II. Transient Analysis

Consider a set of N uniform transmission lines (length d) plus a reference conductor. The propagation along this linear, stationary, passive, reciprocal $2N$-port is described by the generalized telegrapher's equations:

$$-\frac{d}{dz}\mathbf{V}(z,\omega) = [\mathbf{R}(\omega) + j\omega\mathbf{L}(\omega)]\,\mathbf{I}(z,\omega) \qquad (1)$$

$$-\frac{d}{dz}\mathbf{I}(z,\omega) = [\mathbf{G}(\omega) + j\omega\mathbf{C}(\omega)]\,\mathbf{V}(z,\omega) \qquad (2)$$

$\mathbf{R}(\omega), \mathbf{G}(\omega), \mathbf{L}(\omega)$ and $\mathbf{C}(\omega)$ are the frequency dependent resistance, conductance, inductance and capacitance matrices, respectively. In [4]–[5], the link between the electromagnetic models and the corresponding circuit models is discussed in detail. The frequency dependent characteristic impedance matrix $\mathbf{Z}c(\omega)$ of the transmission line structure is defined as:

$$\mathbf{Z}c(\omega) = \{[\mathbf{R}+j\omega\mathbf{L}][\mathbf{G}+j\omega\mathbf{C}]\}^{-0.5}[\mathbf{R}+j\omega\mathbf{L}] \qquad (3)$$

This complex N by N matrix can be considered as the input impedance matrix of the infinitely long coupled transmission line structure.

Manuscript received July 21, 1993; revised March 20, 1994. This paper was recommended by Associate Editor Mamoru Tanaka.

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IEEE Log Number 9403409.

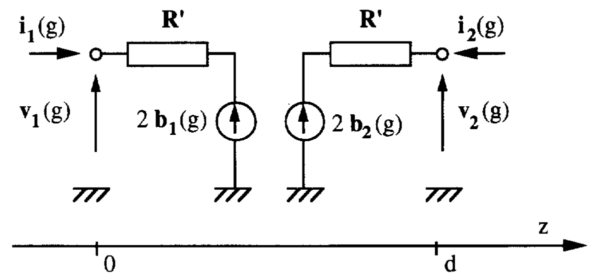

Fig. 1. Extended Thevenin model of transmission line at time $g\Delta t$.

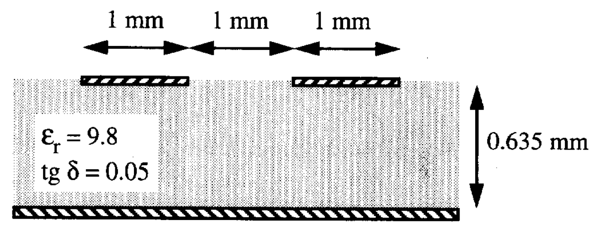

Fig. 2. Cross-section of a symmetric two-line microstrip configuration.

Now, we calculate the Extended Scattering Matrix $\mathbf{S}^{\mathrm{ESM}}$ of the coupled transmission lines [3]. The Extended Scattering Matrix is the S-matrix with reference impedance matrix $\mathbf{R}'$, which is the instantaneous characteristic impedance matrix:

$$\mathbf{R}' = \lim_{\omega\to\infty}\{\mathbf{Z}c(\omega)\} \qquad (4)$$

Note that $\mathbf{S}_{11}^{\mathrm{ESM}} = \mathbf{S}_{22}^{\mathrm{ESM}}$ and $\mathbf{S}_{12}^{\mathrm{ESM}} = \mathbf{S}_{21}^{\mathrm{ESM}}$ due to the symmetry and the reciprocity of the $2N$-port. Based on (1) and (2), $\mathbf{S}_{12}^{\mathrm{ESM}}$ and $\mathbf{S}_{11}^{\mathrm{ESM}}$ are defined as:

$$\begin{aligned}\mathbf{S}_{12}^{\mathrm{ESM}}(\omega) = 2\big[&\mathbf{M_v}\cosh(\Gamma d)\mathbf{M}_v^{-1} + \mathbf{TM_v}\cosh(\Gamma d)\mathbf{M_v}^{-1}\mathbf{T}^{-1}\\ &+ \mathbf{TM_v}\sinh(\Gamma d)\mathbf{M_v}^{-1}\\ &+ \mathbf{M_v}\sinh(\Gamma d)\mathbf{M_v}^{-1}\mathbf{T}^{-1}\big]^{-1}\end{aligned} \qquad (5)$$

$$\begin{aligned}\mathbf{S}_{11}^{\mathrm{ESM}}(\omega) = -0.5\ \mathbf{S}_{12}^{\mathrm{ESM}}(\omega)\big[&\mathbf{M_v}\cosh(\Gamma d)\mathbf{M_v}^{-1}\\ &- \mathbf{TM_v}\cosh(\Gamma d)\mathbf{M_v}^{-1}\mathbf{T}^{-1}\\ &+ \mathbf{TM_v}\sinh(\Gamma d)\mathbf{M_v}^{-1}\\ &- \mathbf{M_v}\sinh(\Gamma d)\mathbf{M_v}^{-1}\mathbf{T}^{-1}\big]\end{aligned} \qquad (6)$$

where:

$$\mathbf{T} = \mathbf{R}'\mathbf{Z}_c(\omega)^{-1} \qquad (7)$$

and where $\Gamma(\omega)$ and $\mathbf{M_v}(\omega)$ are, respectively, the diagonal eigenvalue matrix and the eigenvector matrix of the following eigenvalue problem:

$$\{[\mathbf{R}+j\omega\mathbf{L}][\mathbf{G}+j\omega\mathbf{C}]\}\mathbf{M}_v(\omega) = \mathbf{M}_v(\omega)\Gamma(\omega)^2 \qquad (8)$$

$\mathbf{S}_{11}^{\mathrm{ESM}}(\omega)$ can be considered as a scattering reflection coefficient matrix, while $\mathbf{S}_{12}^{\mathrm{ESM}}(\omega)$ can be seen as a generalized scattering transmission coefficient matrix.

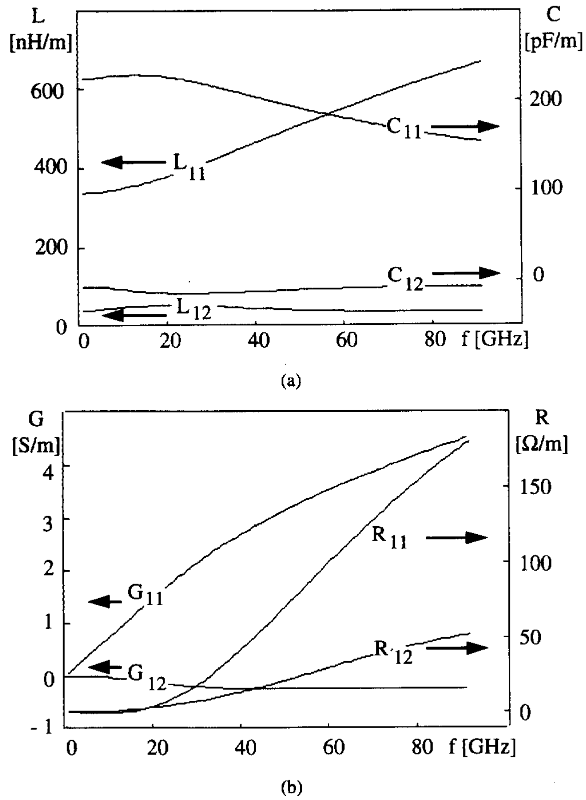

Fig. 3. Frequency dependent transmission line parameter matrices: (a) inductance matrix $\mathbf{L}(\omega)$ and capacitance matrix $\mathbf{C}(\omega)$, (b) conductance matrix $\mathbf{G}(\omega)$ and resistance matrix $\mathbf{R}(\omega)$.

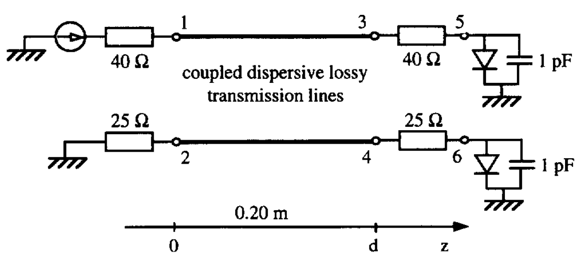

Fig. 4. Coupled dispersive transmission line network with nonlinear loads.

The voltage vector $\mathbf{V_p}(\omega)$ at side p of the transmission line structure ($p = 1, 2$) is related to the voltage and the current vectors at both sides:

$$\mathbf{V}_p(\omega) = \mathbf{R}'\mathbf{I}_p(\omega) + \sum_{q=1}^{2} \mathbf{S}_{\mathbf{pq}}^{\mathrm{ESM}}(\omega)\left[\mathbf{V_q}(\omega) + \mathbf{R}'\mathbf{I_q}(\omega)\right] \quad (9)$$

The reference direction of $\mathbf{I_p}(\omega)$ is pointed towards the p^{th} port of the transmission line structure. The Fast Fourier Transform is used to calculate the time domain formulation. All frequency domain data are bandlimited before transformation in order to avoid aliasing errors. Thanks to the particular choice of the reference matrix $\mathbf{R}'$, the impulse functions $\mathbf{s}_{\mathbf{pp}}^{\mathrm{ESM}}(t)$ and $\mathbf{s}_{\mathbf{pq}}^{\mathrm{ESM}}(t)$ have a short time-duration and they are equal to zero at $t = 0$. Short impulse responses are important for the stability and accuracy of the simulation method. They facilitate the inverse transformation problem, increase the computational speed of the algorithm and decrease the memory requirements and the cumulated errors. In the (frequency independent) lossless case, $\mathbf{s}_{\mathbf{pp}}^{\mathrm{ESM}}(t)$ is zero, and $\mathbf{s}_{\mathbf{pq}}^{\mathrm{ESM}}(t)$ consists of N delayed impulse functions. At each discrete point in time $g\Delta t (g = 0, 1, \ldots)$, both sides of the transmission line structure are reduced to an extended Thevenin circuit model (see Fig. 1), and are described by:

$$\begin{aligned}\mathbf{v_p}(g) &= \mathbf{R}'\mathbf{i}_p(g) \\ &\quad + \sum_{q=1}^{2}\sum_{h=1}^{g} \mathbf{s}_{\mathbf{pq}}^{\mathrm{ESM}}(h)\left[\mathbf{v_q}(g-h) + \mathbf{R}'\mathbf{i_q}(g-h)\right] \\ &= \mathbf{R}'\mathbf{i_p}(g) + 2\mathbf{b_p}(g) \qquad (10)\end{aligned}$$

The Thevenin circuit models are implemented in a new SPICE-like transient simulator.

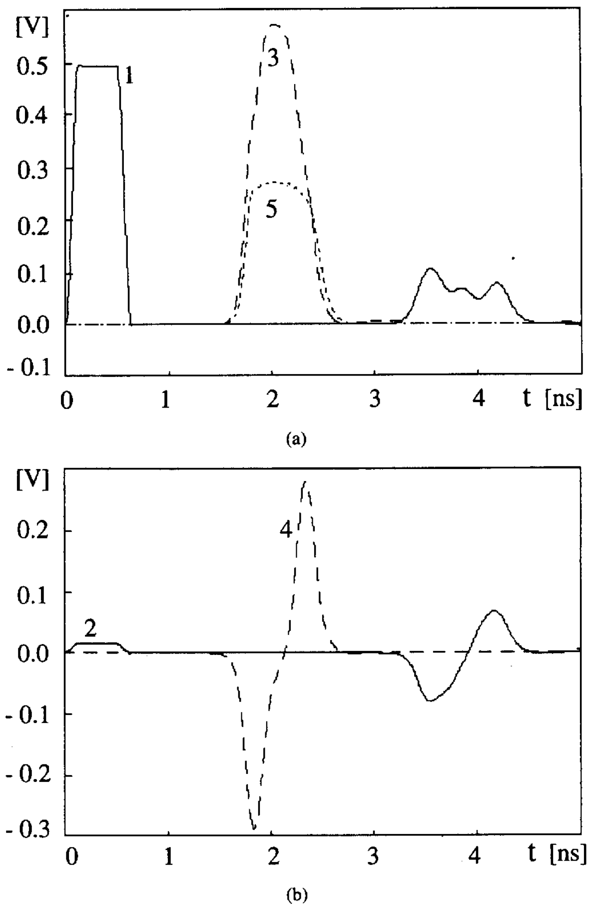

Fig. 5. Responses of the transmission line network of Fig. 4: (a) voltages on active line, (b) voltages on passive line.

III. Example: Uniform Coupled Lossy Microstrip Lines

Consider a symmetric two-line microstrip configuration supported by a lossy substrate (Fig. 2). The frequency dependent line parameter matrices are calculated [6] in the frequency range 0 Hz–100 GHz (Fig. 3). The conductors are assumed to be perfectly conducting. The dielectric losses are important and are taken into account in an exact way. The full-wave line parameter matrices ($\mathbf{R}(\omega), \mathbf{G}(\omega), \mathbf{L}(\omega)$ and $\mathbf{C}(\omega)$) are used to calculate the extended scattering parameters of the four-port transmission structure.

The coupled transmission line network is shown in Fig. 4. The source generates a voltage pulse with an amplitude of 1 V and a width of 500 ps at its half amplitude (2 Gbit/s). The (10%–90%) rise and fall time is 80 ps. The internal impedance of the generator is 40 Ω. At the load side, both lines are terminated in a series combination of a resistor and diode which is shunted by a capacitor of 1 pF. The relation between the current through and the voltage over the diode is given by:

$$i = I_A\left(\exp\left(\frac{v}{V_T}\right) - 1\right) \tag{11}$$

where $I_A = 10$ nA and $V_T = 20$ mV.

The transient simulation results are represented in Fig. 5. Fig. 5(a) shows the voltages on the activated line, while Fig. 5(b) shows the voltages on the passive line. The full (dashed-) lines represent the voltage at the left (right) hand side of the transmission line structure, while the dotted lines show the voltage over the diodes. If the threshold voltage of the diode is reached, the diode starts conducting and changes from an open to a short circuit. Then the reflections are much higher. Note that the transmitted signals are spread out owing to the dispersion.

The nonlinear elements were linearized with an iteration scheme based on the Newton-Raphson algorithm. The simulation time step was chosen to be 8 ps. We calculated 1000 steps in time. The CPU time used was 77 s on a DEC-3100 workstation.

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Real-Time Computation of the Eigenvector Corresponding to the Smallest Eigenvalue of a Positive Definite Matrix

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***Abstract*— In the real-time applications of many signal processing algorithms, such as the TLS (Total Least Squares) algorithm, it is desired to compute as fast as possible the eigenvector corresponding to the smallest eigenvalue of a positive definite matrix. This paper proposes an analog circuit approach to computing in real-time the desired eigenvector. We show analytically and by simulations that the proposed circuit is guaranteed to be stable and to provide the results arbitrarily close to the accurate eigenvector within an elapsed time of only a few characteristic time constants of the circuit.**

I. Introduction

The computation of the eigenvector corresponding to the smallest eigenvalue of a positive definite matrix is a significant problem in many fields of signal processing, such as the TLS (Total Least Squares) frequency estimation [1], bearing estimation [2], and beam forming [3]. In these real-time applications of signal processing, it is desired to compute as fast as possible the eigenvector corresponding to the smallest eigenvalue of a positive definite matrix. However, it is very difficult to compute the desired eigenvector in real-time, mainly because of its high computational complexity. Although some methods to decrease the computational complexity of computing the desired eigenvector have been proposed [1], [4], it is still difficult to deliver the desired real-time performance.

As an alternative, this paper proposes an analog computational model for computing in real-time the eigenvector corresponding to the smallest eigenvalue of a positive definite matrix. As shown in [5], the key features of the analog computational model are asynchronous parallel processing, continuous-time dynamics, and high-speed computational capability. After establishing the stability of the proposed circuit by defining a function, we prove analytically and by simulations that the output of the proposed circuit converges to the eigenvector corresponding to the smallest eigenvalue of a positive definite matrix. In addition, the parameters of the circuit can be obtained from the given matrix without any computations. As a result, this proposed approach is satisfactory for the real-time signal processing.

II. The Architecture and Stability of the Proposed Circuit

The architecture of the circuit proposed in this paper is shown in Fig. 1. In terms of Kirchoff's laws, we have the following dynamic equations for representing the circuit:

$$C_i\frac{dU_i(t)}{dt} = \varphi(t)V_i(t) - \psi(t)\sum_{j=1}^{n}D_{ij}V_j(t) \quad \text{for } i = 1, 2, \cdots, n \tag{1}$$

where the various quantities are described as follows:

$V_i(t)$ and $U_i(t)$ stand for the output and input voltages of the i'th element, respectively.

Manuscript received June 23, 1993; revised December 30, 1993. This work was supported by the Climbing Programme-National Key Project for Fundamental Research in China (Grant NSC92097). This paper was recommended by Associate Editor Heinz Unbehauen.

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IEEE Log Number 9401425.

1057-7122/94$04.00