Analog Integrated Circuits and Signal Processing 5, 57–66 (1994)

Circuit Modeling of General Hybrid Uniform Waveguide Structures in Chiral Anisotropic Inhomogeneous Media

TOM DHAENE, FRANK OLYSLAGER, AND DANIËL DE ZUTTER
Laboratory of Electromagnetism and Acoustics, University of Ghent, Sint-Pietersnieuwstraat 41, 9000 Ghent, Belgium

Received October 8, 1992; Revised May 3, 1993.

Editor:

Abstract. A new generalized high-frequency transmission line model is presented for general uniform hybrid waveguide structures consisting of inhomogeneous isotropic or *anisotropic* media. Starting from Maxwell's equations, the generalized coupled telegrapher's equations are found and the frequency-dependent transmission line parameter matrices $\mathbf{R}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{L}(\omega)$, and $\mathbf{C}(\omega)$ are defined in an unambiguous way as integral expressions of the normalized electromagnetic fields over the cross section of the structure. Some examples illustrate this general circuit-oriented modeling approach. In the appendix we also present a new extended high-frequency circuit model for general uniform hybrid waveguide structures consisting of *chiral* anisotropic inhomogeneous media.

1. Introduction

The propagation behavior and the circuit modeling of conductors embedded in homogeneous isotropic media was studied by several authors. Originally, the circuit modeling was based on quasi-TEM approximations [1, 2]. More recently, the hybrid nature of multilayered interconnection structures is taken into account. A frequency-dependent circuit representation is necessary to model the dispersive nature of the hybrid waveguide. The single hybrid line in an isotropic medium is extensively analyzed in [3] and [4]. In [5–10], this high-frequency modeling approach is extended to analyze general uniform coupled lossy open dispersive waveguide structures in isotropic media.

The circuit modeling of waveguide structures consisting of anisotropic media (such as boron nitride or sapphire substrates) is seldom discussed in literature [11, 12]. In this contribution we present a new generalized high-frequency transmission line model for arbitrary coupled lossy open dispersive waveguide structures embedded in inhomogeneous isotropic or *anisotropic* media. The power-current (*PI*) formulation is used for stripline or microstrip-like structures while the power-voltage (*PV*) formulation is used for coplanar structures. The general frequency-dependent telegrapher's equations proceed directly from Maxwell's equations and the transmission line parameters $\mathbf{R}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{L}(\omega)$, and $\mathbf{C}(\omega)$ are defined in an unambiguous way as integrals of the normalized electromagnetic fields over the cross section of the structure. By unambiguous we mean that $\mathbf{R}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{L}(\omega)$, and $\mathbf{C}(\omega)$ are defined uniquely as soon as a formulation, i.e., *PI* or *PV*, is chosen with a definition for I or V. These transmission line parameter matrices can easily be used in high-frequency network simulators [13, 14]. The matrix formalism is used throughout this study. This guarantees a compact and very general circuit description which is well suited for transient simulation and for CAD and CAE applications. Some examples illustrate this new general circuit-oriented modeling approach. Furthermore, in the appendix a new extended high-frequency transmission line model is proposed for general uniform hybrid waveguide structures consisting of *chiral* anisotropic inhomogeneous media.

2. Normalized Electromagnetic Fields

In the sequel, we will use the phasor notation and the common time dependence $\exp(j\omega t)$ will be omitted.

Consider a uniform coupled lossy dispersive waveguide structure consisting of $N + 1$ conductors with arbitrary cross section embedded in an inhomogeneous anisotropic medium (see figure 1). The z-coordinate is used to denote the longitudinal or propagation direction,

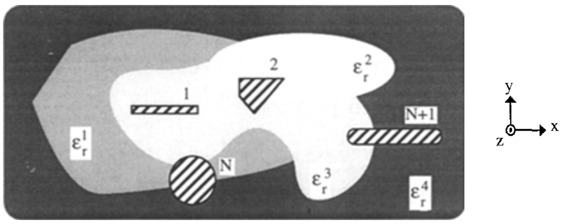

Fig. 1. General uniform waveguide structure with $N + 1$ conductors.

while the x- and y-coordinates are used to denote the transversal direction. The $(N + 1)$th conductor is chosen as the reference or ground conductor. The 3 by 3 complex permittivity $\bar{\epsilon}(x, y)$ and the complex permeability $\bar{\mu}(x, y)$ tensors of the multilayered media are respectively

$$\bar{\epsilon}(x, y) = \begin{bmatrix} \epsilon_0 \bar{\epsilon}_{rt}(x, y) + \dfrac{\bar{\sigma}_t(x, y)}{j\omega} & 0 \\ 0 & \epsilon_0 \epsilon_{rl}(x, y) + \dfrac{\sigma_l(x, y)}{j\omega} \end{bmatrix} = \begin{bmatrix} \bar{\epsilon}_t(x, y) & 0 \\ 0 & \epsilon_l(x, y) \end{bmatrix} \quad (1.a)$$

and

$$\bar{\mu}(x, y) = \begin{bmatrix} \bar{\mu}_t^R(x, y) + j\bar{\mu}_t^I(x, y) & 0 \\ 0 & \mu_l^R(x, y) + j\mu_l^I(x, y) \end{bmatrix} = \begin{bmatrix} \bar{\mu}_t(x, y) & 0 \\ 0 & \mu_l(x, y) \end{bmatrix} \quad (1.b)$$

Hence we allow full anisotropy in the cross section of the structure. The bar on top of a symbol represents either a tensor (such as $\bar{\epsilon}$) or a vector (such as the electric field $\bar{E}$) consisting of spatial components. $\bar{\epsilon}_t$ and $\bar{\mu}_t$ are 2 by 2 tensors.

Now, we focus our attention on the N fundamental propagating modes of the $N + 1$ conductor structure. The fields associated with each mode are written as the sum of a longitudinal z-component and transversal (x, y)-components [8]. The global electromagnetic field propagating in the z-direction is the sum of the partial fields of the N fundamental modes:

$$\bar{E}(x, y, z) = \mathbf{V_M}(z)^T \bar{\mathbf{E}}_t^{\mathbf{M}}(x, y) + R_0 \mathbf{I_M}(z)^T \bar{\mathbf{E}}_l^{\mathbf{M}}(x, y) \quad (2.a)$$

$$\bar{H}(x, y, z) = \mathbf{I_M}(z)^T \bar{\mathbf{H}}_t^{\mathbf{M}}(x, y) + \frac{1}{R_0} \mathbf{V_M}(z)^T \bar{\mathbf{H}}_l^{\mathbf{M}}(x, y) \quad (2.b)$$

R_0 is the characteristic impedance of free-space. The superscript T is used to denote the transposition operator. The boldfaced symbols represent matrices. The subscript t indicates transversal components and the subscript l indicates the longitudinal component. The superscript M indicates modal quantities while the superscript c will be reserved for circuit quantities. Hence, $\bar{\mathbf{E}}_t^{\mathbf{M}}$ is a column matrix consisting of N modal space vectors $\bar{E}_{t,i}^M$ $(i = 1, \ldots, N)$ with two space components $\bar{E}_{t,i}^{M,x}$ and $\bar{E}_{t,i}^{M,y}$. The N by 1 column matrices $\mathbf{V_M}$ and I_M represent the modal voltages and currents respectively. They are not independent. If we consider a mode i propagating in the positive (negative) z-direction, we have that $V_{M,i} = (-)Z_{M,i} I_{M,i}$. $\mathbf{Z_M}$ is the diagonal N by N modal impedance matrix.

We restrict ourselves to the N fundamental modes because we want to represent the waveguide by a system of N coupled transmission lines. It is possible to take into account higher-order modes and represent the waveguide by more than N coupled transmission lines. In this last case a special conversion network has to be inserted between the transmission line and the load and generator, because the load and generator have only N wires (+ ground).

In order to simulate a general hybrid interconnection structure with a circuit simulator we will transform the modal description into a circuit model consisting of coupled lossy dispersive transmission lines. The circuit voltage vector $\mathbf{V_c}(z)$ and the circuit current vector $\mathbf{I_c}(z)$ are

$$\mathbf{I_c}(z) = \mathbf{M_I} \mathbf{I_M}(z) \quad (3.a)$$

$$\mathbf{V_c}(z) = \mathbf{M_V} \mathbf{V_M}(z) \quad (3.b)$$

where $\mathbf{M_I}$ and $\mathbf{M_V}$ are frequency dependent N by N transformation matrices between the modal and the circuit quantities [8]. These transformation matrices follow from the requirement that the circuit model and the real waveguide structure must have the same diagonal N by N modal propagation matrix $\mathbf{\Gamma}$ and that they must propagate the same average complex power. However, these two conditions do not define $\mathbf{M_I}$ and $\mathbf{M_V}$ in a unique way. The remaining degree of freedom can be used to obtain a "quasi-TEM equivalence" between the circuit model and the real waveguide structure. For microstrips and striplines for example, the *PI* model is well suited as a circuit description. In that case the circuit current associated with each conductor is chosen to be identical to the total longitudinal current flowing on each conductor. On the other hand, for slotlines and coplanar interconnection structures, the *PV* model is most suited for circuit simulation. Then, the voltage is defined as a line integral of the electric field along a suitable integration path. Only in the quasi-TEM limit, i.e., for low frequencies, both circuit parameters, voltage and current, have a unique and clear circuit interpretation [10]. This is because the fundamental modes become nonhybrid at low frequencies, i.e, they lose their longitudinal field components.

In (2), the global electromagnetic fields are represented as a sum of partial modal fields. With (3) the electromagnetic fields (2) can be expressed as a function of circuit-related parameters:

$$\bar{E}(x, y, z) = \mathbf{V}_c(z)^T\bar{\mathbf{E}}_t^c(x, y) + R_0\mathbf{I}_c(z)^T\bar{\mathbf{E}}_l^c(x, y) \tag{4.a}$$

$$\bar{H}(x, y, z) = \mathbf{I}_c(z)^T\bar{\mathbf{H}}_t^c(x, y) + \frac{1}{R_0}\mathbf{V}_c(z)^T\bar{\mathbf{H}}_l^c(x, y) \tag{4.b}$$

where

$$\begin{aligned}
\bar{\mathbf{E}}_t^c(x, y) &= (\mathbf{M}_V^T)^{-1}\bar{\mathbf{E}}_t^M(x, y),\\
\bar{\mathbf{E}}_l^c(x, y) &= (\mathbf{M}_I^T)^{-1}\bar{\mathbf{E}}_l^M(x, y)\\
\bar{\mathbf{H}}_t^c(x, y) &= (\mathbf{M}_I^T)^{-1}\bar{\mathbf{H}}_t^M(x, y),\\
\bar{\mathbf{H}}_l^c(x, y) &= (\mathbf{M}_V^T)^{-1}\bar{\mathbf{H}}_l^M(x, y)
\end{aligned} \tag{5}$$

The N by 1 column matrices $\mathbf{V}_c$ and $\mathbf{I}_c$ represent the circuit voltages and currents respectively. With each conductor corresponds a unique transversal and longitudinal normalized electric and magnetic field. These normalized fields can be seen as a weighted sum of the propagating modal fields.

3. Generalized Transmission Line Equations

The total electromagnetic field (at angular frequency ω) consists of the sum of the normalized partial fields as can be seen in (4). We substitute these expressions in Maxwell's equations, and separate the longitudinal and the transversal components. In this way we find the generalized transmission line equations which describe the propagation behavior in coupled lossy open dispersive waveguide structures in isotropic or anisotropic inhomogeneous media:

$$-\frac{d}{dz}\mathbf{V}_c(z) = \mathbf{Z}_{cir}\mathbf{I}_c(z) \tag{6.a}$$

$$-\frac{d}{dz}\mathbf{I}_c(z) = \mathbf{Y}_{cir}\mathbf{V}_c(z) \tag{6.b}$$

with

$$\mathbf{Z}_{cir}(\omega) = \mathbf{R}(\omega) + j\omega\mathbf{L}(\omega) = \mathbf{M}_V\Gamma\mathbf{Z}_M\mathbf{M}_I^{-1} \tag{7.a}$$

$$\mathbf{Y}_{cir}(\omega) = \mathbf{G}(\omega) + j\omega\mathbf{C}(\omega) = \mathbf{M}_I\Gamma\mathbf{Z}_M^{-1}\mathbf{M}_V^{-1} \tag{7.b}$$

where $\mathbf{Z}_{cir}(\omega)$ and $\mathbf{Y}_{cir}(\omega)$ are the circuit impedance and the circuit admittance line matrices per unit length. $\mathbf{R}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{L}(\omega)$, and $\mathbf{C}(\omega)$ are respectively the generalized resistance, capacitance, conductance, and inductance matrices per unit length. This new transmission line circuit model is fully compatible with, and is an extension towards higher frequencies of the well-known TEM and quasi-TEM circuit models.

Maxwell's equations also lead to the following vector relations between the longitudinal and the transversal normalized electromagnetic fields:

$$\nabla_t \times \bar{\mathbf{E}}_t^c(x, y) + j\omega\frac{\mu_l(x, y)}{R_0}\bar{\mathbf{H}}_l^c(x, y) = 0 \tag{8.a}$$

$$\mathbf{Z}_{cir}^T\bar{1}_z \times \bar{\mathbf{E}}_t^c(x, y) - R_0\nabla_t \times \bar{\mathbf{E}}_l^c(x, y) - j\omega\bar{\bar{\mu}}_t(x, y)\bar{\mathbf{H}}_t^c(x, y) = 0 \tag{8.b}$$

$$\nabla_t \times \bar{\mathbf{H}}_t^c(x, y) - j\omega\epsilon_l(x, y)R_0\bar{\mathbf{E}}_l^c(x, y) = 0 \tag{9.a}$$

$$\mathbf{Y}_{cir}^T\bar{1}_z \times \bar{\mathbf{H}}_t^c(x, y) - \frac{\nabla_t \times \bar{\mathbf{H}}_l^c(x, y)}{R_0} + j\omega\bar{\bar{\epsilon}}_t(x, y)\bar{\mathbf{E}}_t^c(x, y) = 0 \tag{9.b}$$

$$\nabla_t \cdot \bar{\bar{\epsilon}}_t(x, y)\bar{\mathbf{E}}_t^c(x, y) - R_0\epsilon_l(x, y)\mathbf{Y}_{cir}^T\bar{1}_z \cdot \bar{\mathbf{E}}_l^c(x, y) = 0 \tag{10}$$

$$\nabla_t \cdot \bar{\bar{\mu}}_t(x, y)\bar{\mathbf{H}}_t^c(x, y) - j\frac{\mu_l(x, y)}{R_0}\mathbf{Z}_{cir}^T\bar{1}_z \cdot \bar{\mathbf{H}}_l^c(x, y) = 0 \tag{11}$$

The results in [3] can be seen as a special case (isotropic medium and $N = 1$) of the general equations (8)–(11).

Elimination of the longitudinal fields in (8)–(11) leads to two eigenvalue equations with the normalized transversal fields as eigenvectors. The electric eigenvalue equation is found to be

$$\begin{aligned}
&[-(\Lambda_V^T)^2 + \omega^2\bar{1}_z \times \bar{\bar{\mu}}_t\bar{1}_z \times \bar{\bar{\epsilon}}_t]\bar{\mathbf{E}}_t^c(x, y)\\
&= \bar{1}_z \times \bar{\bar{\mu}}_t\bar{1}_z \times \nabla_t \times \left[\frac{1}{\mu_l}\nabla_t \times \bar{\mathbf{E}}_t^c(x, y)\right]\\
&+ \nabla_t\left[\frac{1}{\epsilon_l}\nabla_t \cdot \bar{\bar{\epsilon}}_t\bar{\mathbf{E}}_t^c(x, y)\right]
\end{aligned} \tag{12}$$

and the magnetic eigenvalue equation is

$$[-(\Lambda_I^T)^2 + \omega^2 \bar{1}_z \times \bar{\epsilon}_t \bar{1}_z \times \bar{\mu}_t]\bar{\mathbf{H}}_t^c(x, y) = \bar{1}_z \times \bar{\epsilon}_t \bar{1}_z \times \nabla_t \times \left[\frac{1}{\epsilon_l} \nabla_t \times \bar{\mathbf{H}}_t^c(x, y)\right] + \nabla_t \left[\frac{1}{\mu_l} \nabla_t \cdot \bar{\mu}_t \bar{\mathbf{H}}_t^c(x, y)\right] \quad (13)$$

The eigenvalue matrices $\Lambda_V(\omega)$ $(=\mathbf{M}_V \Gamma \mathbf{M}_V^{-1})$ and $\Lambda_I(\omega)$ $(=\mathbf{M}_I \Gamma \mathbf{M}_I^{-1})$ represent the complex voltage and current propagation matrix respectively. They are defined in a unique way. Usually, these matrices are not symmetric.

4. Transmission Line Parameters Matrices

In [3], the line parameters R, G, L, and C of a single transmission line in an isotropic multilayered medium are calculated based on Maxwell's equations. Now, we search for a physical interpretation of the transmission line parameter matrices $\mathbf{R}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{L}(\omega)$, and $\mathbf{C}(\omega)$ for the N fundamental modes of the general interconnection structure under study in an analogous way as in [3].

Based on the fact that both the waveguide structure and the equivalent circuit model must propagate the same complex average power, we can prove [8] that the partial transversal electromagnetic fields $\bar{E}_{t,i}^c(x, y)$ and $\bar{H}_{t,j}^c(x, y)$, associated with the conductors i and j, are power orthonormal, i.e.,

$$\mathbf{1} = \int_S \int [\bar{\mathbf{E}}_t^c(x, y) \times \bar{\mathbf{H}}_t^c(x, y)^{*T}] \cdot \bar{1}_z \, dS \quad (14)$$

where S represents the cross section of the waveguide structure.

We now substitute Maxwell's equations (8.b) and (9.a) into (14) and integrate by parts, assuming that the fields vanish at the boundaries of the waveguide (or at infinity). This leads to the N by N resistance and inductance matrices:

$$\mathbf{R}(\omega) + j\omega\mathbf{L}(\omega) = \int_S \int j\omega[- R_0^2 \epsilon_l^*(x, y)\bar{\mathbf{E}}_l^c(x, y)^* \cdot \bar{\mathbf{E}}_l^c(x, y)^T + \bar{\mu}_t(x, y)^T \bar{\mathbf{H}}_t^c(x, y)^* \cdot \bar{\mathbf{H}}_t^c(x, y)^T] \, dS \quad (15)$$

The presence of longitudinal electric fields influences the elements of the resistance and the inductance matrix as compared to the quasi-static limit, in which case the longitudinal components are negligible.

Starting from (14) and from Maxwell's equations (8)–(11), the N by N conductance and capacitance matrices are found in an analogous way as above:

$$\mathbf{G}(\omega) + j\omega\mathbf{C}(\omega) = \int_S \int j\omega[\bar{\epsilon}_t(x, y)^T \bar{\mathbf{E}}_t^c(x, y)^* \cdot \bar{\mathbf{E}}_t^c(x, y)^T - \frac{\mu_l^*(x, y)}{R_0^2} \bar{\mathbf{H}}_l^c(x, y)^* \cdot \bar{\mathbf{H}}_l^c(x, y)^T] \, dS \quad (16)$$

Now the longitudinal magnetic fields influence the value of the capacitance and conductance matrix elements. Due to the presence of these longitudinal magnetic fields, the high-frequency conductance matrix $\mathbf{G}(\omega)$ cannot be found by simply replacing $\epsilon_0 \bar{\epsilon}_{rt}(x, y)$ by $\bar{\sigma}_t(x, y)/j\omega$ in the calculation of the capacitance matrix $\mathbf{C}(\omega)$.

The definitions of the line parameter matrices are based on Maxwell's equations and on the equivalence of the propagated power in the real waveguide and the transmission line model. At high frequencies and when losses are important, the line parameter matrices $\mathbf{R}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{L}(\omega)$, and $\mathbf{C}(\omega)$ given above may differ from the usual static ones or from the ones obtained by applying a perturbation analysis for small losses.

Based on these line parameter matrices, the frequency-dependent characteristic impedance matrix $\mathbf{Z}_c(\omega)$ can be defined in an unambiguous way:

$$\mathbf{Z}_c(\omega) = \{[\mathbf{R} + j\omega\mathbf{L}][\mathbf{G} + j\omega\mathbf{C}]\}^{-0.5}[\mathbf{R} + j\omega\mathbf{L}] = \mathbf{M}_V \mathbf{M}_I^{-1} \quad (17)$$

The characteristic impedance matrix relates the circuit current waves to the circuit voltage waves propagating in the positive z-direction and can be seen as the input impedance matrix of the infinitely long coupled transmission line structure.

In Appendix A the propagation behavior of the fundamental modes of a general uniform coupled dispersive open hybrid waveguide structure in a *chiral anisotropic* inhomogeneous medium is studied, and an extended high-frequency transmission line is presented. Some results of previous studies [1–10] can be seen as a special case (quasi-static approximation, single line, lossless case, isotropic medium) of this new more universal approach.

5. Numerical Examples

5.1. Asymmetric Two-Line Microstrip Configuration (Isotropic)

Consider an asymmetric two-line microstrip configuration laying on an isotropic lossy substrate. The cross section of the strip configuration is shown in figure 2. The structure consists of a perfectly conducting ground plane, a lossy dielectric substrate (tan δ = 0.05, ϵ_r = 9.8), and a semi-infinite air top layer. The strips are infinitely thin and perfectly conducting. Two fundamental modes can propagate in this two-line system: a c-mode and a π-mode. Using a new rigorous full-wave integral equation technique [15], the frequency dependent modal propagation factors γ_i ($=j\beta_i$) (i = 1, 2) and the line-mode characteristic impedances are calculated in the frequency range 0–100 GHz (figure 3). All losses are included in an exact way without making any approximations or perturbations. The integral equation technique is an integral formulation of (12) and (13) and delivers the propagation constants of the eigenmodes. The technique is also extended to calculate products of the field components, as in (15) and (16), over the cross section of the structure. In figure 4, the elements of the $\mathbf{L}(\omega)$, $\mathbf{G}(\omega)$, and $\mathbf{C}(\omega)$ matrices are shown as a function of frequency. Note the significant frequency dependence of all these parameters. The $\mathbf{R}(\omega)$ matrix is negligible because we only have dielectric losses. As expected, the elements of $\mathbf{G}(\omega)$ are rather high due to the conductance of the dielectric layer (tan δ = 0.05). The electromagnetic fields concentrate more and more in the lossy dielectric layer as the frequency increases. Hence, $L_{11}(\omega)$ and $L_{22}(\omega)$ increase and $C_{11}(\omega)$ and $C_{22}(\omega)$ decrease with frequency. The dielectric losses

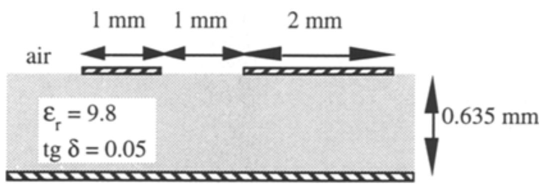

Fig. 2. Cross section of the asymmetric two-line microstrip configuration.

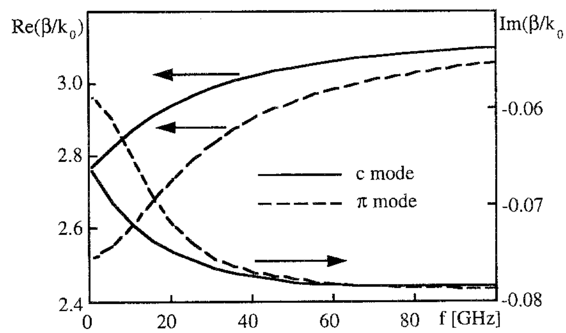

Fig. 3. Frequency-dependent propagation factors of the fundamental modes of the structure of figure 2.

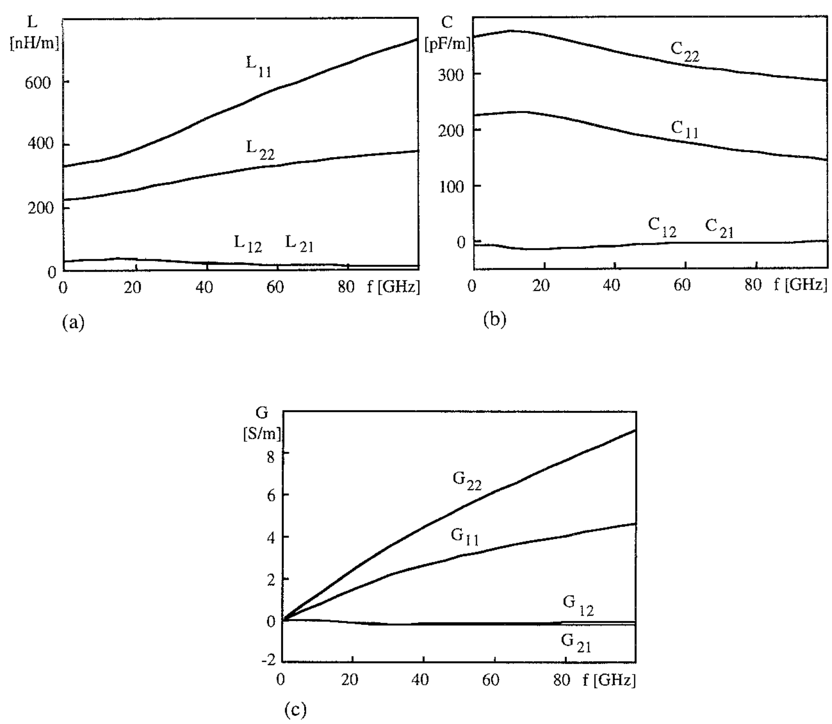

Fig. 4. Frequency-dependent transmission line parameter matrices for the structure of figure 2: (a) inductance matrix $\mathbf{L}(\omega)$; (b) capacitance matrix $\mathbf{C}(\omega)$; (c) conductance matrix $\mathbf{G}(\omega)$.

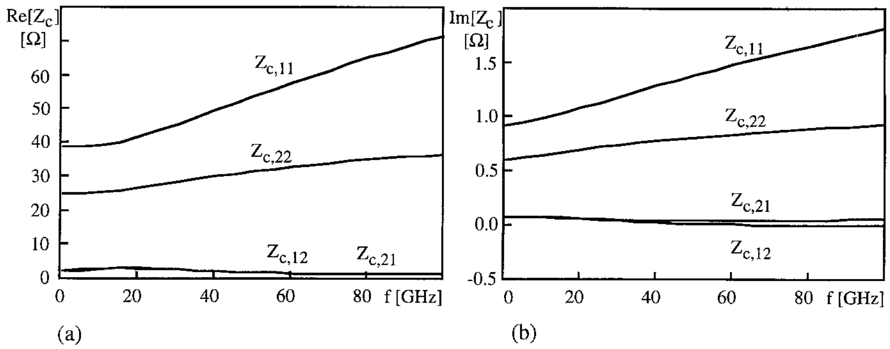

Fig. 5. Frequency-dependent elements of the characteristic impedance matrix $\mathbf{Z}_c(\omega)$ for the structure of figure 2.

increase with frequency, so the absolute values of the elements of $\mathbf{G}(\omega)$ will also increase with frequency. In figure 5 the elements of the complex characteristic impedance matrix $\mathbf{Z}_c(\omega)$ are shown.

5.2. Coaxial Cable (Anisotropic)

Figure 6 shows the cross section of a coaxial cable filled with two homogeneous anisotropic dielectrics. The inner and outer conductor (radius a and c) consist of perfectly conducting material. The permittivity $\bar{\epsilon}_i$ and the permeability $\bar{\mu}_i$ tensors of both anisotropic media are given by

$$\bar{\epsilon}_i = \begin{pmatrix} \epsilon_{t,i} & 0 & 0 \\ 0 & \epsilon_{t,i} & 0 \\ 0 & 0 & \epsilon_{l,i} \end{pmatrix} \tag{18.a}$$

and

$$\bar{\mu}_i = \begin{pmatrix} \mu_{t,i} & 0 & 0 \\ 0 & \mu_{t,i} & 0 \\ 0 & 0 & \mu_{l,i} \end{pmatrix} \tag{18.b}$$

where $i = 1, 2$.

We suppose that $\epsilon_{t,1}\mu_{t,1} > \epsilon_{t,2}\mu_{t,2}$. The eigenvalue equation for the propagation factor β of the fundamental mode can be found analytically:

$$\frac{\gamma_{E,1}\epsilon_{t,1}}{\gamma_{t,1}^2}\left[J_0'(\gamma_{E,1}b)Y_0(\gamma_{E,1}a) - Y_0'(\gamma_{E,1}b)J_0(\gamma_{E,1}a)\right] \times \left[I_0(\delta_{E,2}b)K_0(\delta_{E,2}c) - K_0(\delta_{E,2}b)I_0(\delta_{E,2}c)\right]$$

$$= \frac{\delta_{E,2}\epsilon_{t,2}}{\gamma_{t,2}^2}\left[J_0(\gamma_{E,1}b)Y_0(\gamma_{E,1}a) - Y_0(\gamma_{E,1}b)J_0(\gamma_{E,1}a)\right] \times \left[I_0'(\delta_{E,2}b)K_0(\delta_{E,2}c) - K_0'(\delta_{E,2}b)I_0(\delta_{E,2}c)\right] \tag{19}$$

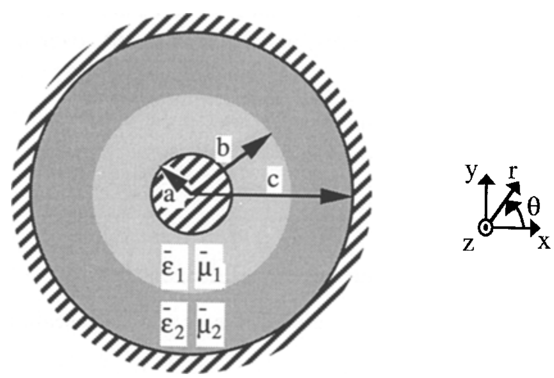

Fig. 6. Cross section of a coaxial cable filled with an inhomogeneous (an)isotropic dielectric.

with

$$\gamma_{t,i}^2 = \omega^2\epsilon_{t,i}\mu_{t,i} - \beta^2 \tag{20.a}$$

$$\gamma_{E,1} = \sqrt{\frac{\epsilon_{l,1}}{\epsilon_{t,1}}\left(\omega^2\epsilon_{t,1}\mu_{t,1} - \beta^2\right)} \tag{20.b}$$

$$\delta_{E,2} = \sqrt{\frac{\epsilon_{l,2}}{\epsilon_{t,2}}\left(\beta^2 - \omega^2\epsilon_{t,2}\mu_{t,2}\right)} \tag{20.c}$$

J_0 respectively Y_0 are the Bessel function of zeroth order and first respectively second kind. I_0 respectively K_0 are the modified Bessel function of zeroth order and first respectively second kind. J_0', Y_0', I_0', and K_0' are the derivatives of J_0, Y_0, I_0, and K_0 respectively.

We studied a coaxial structure filled with anisotropic dielectrics (a = 1 mm, b = 2 mm, c = 4 mm, $\epsilon_{t,1} = 4$, $\epsilon_{l,1} = \epsilon_{l,2} = 2$, $\epsilon_{t,2} = \mu_{t,1} = \mu_{t,2} = \mu_{l,1} = \mu_{l,2} = 1$) and a similar one filled with isotropic dielectrica (a = 1 mm, b = 2 mm, c = 4 mm, $\epsilon_{t,1} = \epsilon_{l,1} = 4$, $\epsilon_{t,2} = \epsilon_{l,2} = \mu_{t,1} = \mu_{t,2} = \mu_{l,1} = \mu_{l,2} = 1$). For both structures, we calculated the propagation factor γ ($=j\beta$)

(figure 7), the transmission line parameters $L(\omega)$ and $C(\omega)$ (figure 8) and the characteristic impedance $Z_c(\omega)$ (figure 9) in the frequency range 0–55 GHz. The full lines represent the anisotropic case, while the dashed lines represent the corresponding isotropic case.

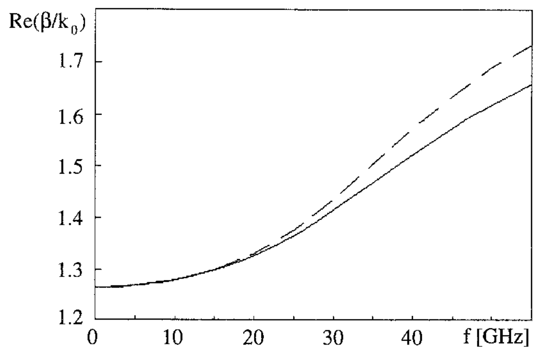

Fig. 7. Frequency-dependent propagation factor of the fundamental mode of the structure of figure 6 (______ : anisotropic/__ __ __: isotropic).

6. Conclusion

A new generalized high-frequency transmission line model has been proposed for general uniform coupled dispersive open waveguides with arbitrary cross sections in isotropic or *anisotropic* multilayered media. This frequency dependent circuit model is well suited for CAD and CAE applications. The *PI* formulation is used for stripline or microstrip-like structures while the *PV* formulation is used for coplanar structures. Based on Maxwell's equations, the generalized coupled telegrapher's equations are found and the frequency-dependent "anisotropic" transmission line parameter matrices $\mathbf{R}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{L}(\omega)$, and $\mathbf{C}(\omega)$ are defined as

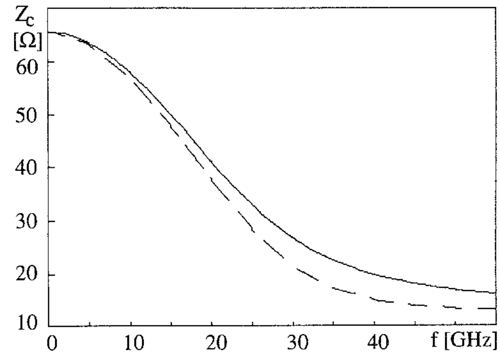

Fig. 9. Frequency-dependent characteristic impedance $Z_c(\omega)$ for the structure of figure 6 (______ : anisotropic/__ __ __: isotropic).

integral expressions of the normalized electromagnetic fields over the cross section of the structure. In the appendix a new extended high-frequency transmission line model is proposed for general uniform hybrid waveguide structures consisting of *chiral anisotropic* media.

Appendix A. Description of a General Uniform Waveguide Structure Containing Chiral Anisotropic Media

Consider a general uniform coupled dispersive open hybrid waveguide structure consisting of $N + 1$ conductors with arbitrary cross section embedded in an inhomogeneous *chiral anisotropic* medium. One line is chosen as reference conductor. N fundamental modes can propagate in this waveguide structure. First we study the propagation behavior of one particular propagating mode. Afterwards we present a circuit model for all fundamental modes.

The electromagnetic fields, which correspond to a fundamental mode propagating in anisotropic chiral

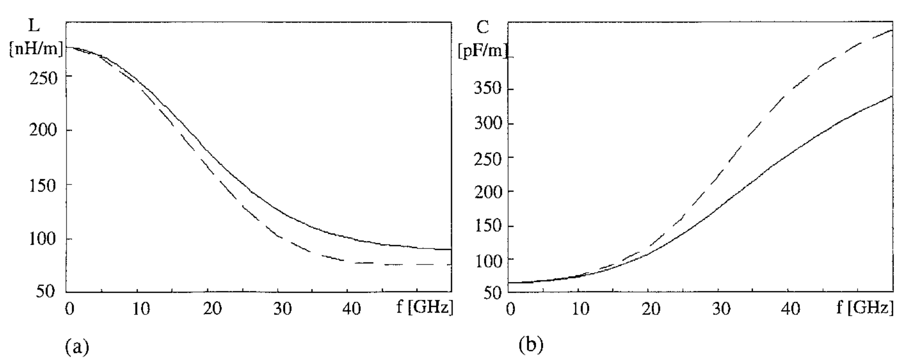

Fig. 8. Frequency-dependent transmission line parameters for the structure of figure 6 (______ : anisotropic/__ __ __: isotropic): (a) inductance parameter $L(\omega)$, (b) capacitance parameter $C(\omega)$.

media, satisfy the following constitutive relations [16–17]:

$$\bar{B} = \bar{\mu}\bar{H} + j\omega\bar{\chi}\bar{E} \tag{A.1.a}$$

$$\bar{D} = \bar{\epsilon}\bar{E} - j\omega\bar{\chi}\bar{H} \tag{A.1.b}$$

where the electric and magnetic field are denoted by $\bar{E}$ and $\bar{H}$, and the electric and magnetic induction by $\bar{D}$ and $\bar{B}$ respectively. $\bar{\chi}$ is the chirality tensor:

$$\bar{\chi}(x, y) = \begin{bmatrix} \bar{\chi}_t(x, y) & 0 \\ 0 & \chi_l(x, y) \end{bmatrix} \tag{A.2}$$

The modal electromagnetic field associated with a propagating hybrid mode can be divided in longitudinal and transversal components:

$$\bar{E}(x, y, z) = V_M(z)\bar{E}_t^M(x, y) + \bar{E}_l(x, y, z) \tag{A.3.a}$$

$$\bar{H}(x, y, z) = I_M(z)\bar{H}_t^M(x, y) + \bar{H}_l(x, y, z) \tag{A.3.b}$$

The longitudinal field components (subindex l) depend on the x-, y-, and z-coordinates, while the transversal field components (subindex t) depend only on the x- and the y-coordinates (transversal coordinates). The parameters V_M and I_M are the modal voltage and the modal current of the fundamental mode under study. Their dimension is volt and ampere respectively. They depend only upon the longitudinal z-coordinate.

Now, based on Maxwell's rotor equations, we define the generalized longitudinal electromagnetic fields $\bar{E}_l^M$ and $\bar{H}_l^M$ which depend only on the x- and y-coordinates (transversal coordinates):

$$\begin{bmatrix} \frac{V_M(z)}{R_0}\bar{H}_l^M(x, y) \\ R_0 I_M(z)\bar{E}_l^M(x, y) \end{bmatrix} = \begin{bmatrix} 1 & \frac{j\omega\chi_l}{\mu_l} \\ -\frac{j\omega\chi_l}{\epsilon_l} & 1 \end{bmatrix} \begin{bmatrix} \bar{H}_l(x, y, z) \\ \bar{E}_l(x, y, z) \end{bmatrix} \tag{A.4}$$

$\bar{E}_l^M$ and $\bar{H}_l^M$ now depend both on $\bar{E}_l$ and $\bar{H}_l$. Substitution of (A.3) and (A.4) in Maxwell's equations, and separation of the longitudinal and the transversal components learns that the modal voltages and currents are now related by the extended modal telegrapher's equations:

$$-\frac{d}{dz}\begin{bmatrix} V_M(z) \\ I_M(z) \end{bmatrix} = \begin{bmatrix} \frac{j\omega R_0\chi_l\nabla_t \cdot \bar{\chi}_t\bar{E}_t^M}{\mu_l\bar{1}_z \cdot \bar{H}_l^M} & \frac{R_0\nabla_t \cdot \bar{\mu}_t\bar{H}_t^M}{\mu_l\bar{1}_z \cdot \bar{H}_l^M} \\ \frac{\nabla_t \cdot \bar{\epsilon}_t\bar{E}_t^M}{R_0\epsilon_l\bar{1}_z \cdot \bar{E}_l^M} & -\frac{j\omega\nabla_t \cdot \bar{\chi}_t\bar{H}_t^M}{R_0\epsilon_l\bar{1}_z \cdot \bar{E}_l^M} \end{bmatrix} \begin{bmatrix} V_M(z) \\ I_M(z) \end{bmatrix} = \begin{bmatrix} U_M & Z_M \\ Y_M & J_M \end{bmatrix} \begin{bmatrix} V_M(z) \\ I_M(z) \end{bmatrix} \tag{A.5}$$

where Z_M and Y_M represent the modal per unit length series impedance and shunt admittance respectively. U_M and J_M represent the modal per unit length series voltage source factor and shunt current source factor. With each fundamental mode corresponds an extended transmission line model. This per unit length model is shown in figure A.1. Equation (A.5) can also be written as

$$-\frac{d}{dz}\begin{bmatrix} V_M(z) \\ I_M(z) \end{bmatrix} = \begin{bmatrix} \frac{\gamma_1 Z_1 - \gamma_2 Z_2}{Z_1 - Z_2} & Z_1 Z_2 \frac{\gamma_2 - \gamma_1}{Z_1 - Z_2} \\ \frac{\gamma_1 - \gamma_2}{Z_1 - Z_2} & \frac{\gamma_2 Z_1 - \gamma_1 Z_2}{Z_1 - Z_2} \end{bmatrix} \begin{bmatrix} V_M(z) \\ I_M(z) \end{bmatrix} \tag{A.6}$$

where γ_1 and γ_2 are modal propagation factors:

$$\gamma_{1,2} = \frac{(U_M + J_M) \pm \sqrt{(U_M - J_M)^2 + 4Z_M Y_M}}{2} \tag{A.7}$$

and Z_1 and Z_2 are modal impedances:

$$Z_{1,2} = \frac{Z_M}{\gamma_{1,2} - U_M} \tag{A.8}$$

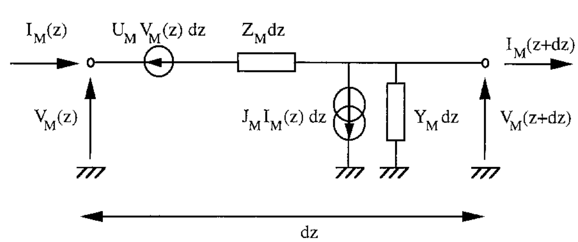

Fig. A.1. Per unit length model of the fundamental mode propagating in an inhomogeneous chiral anisotropic medium.

The modal voltage $V_M(z)$ and current $I_M(z)$ are the sum of wave components propagating in the negative and positive z-direction:

$$V_M(z) = V_M^1(0)e^{-\gamma_1 z} + V_M^2(0)e^{-\gamma_2 z} \quad \text{(A.9.a)}$$

$$I_M(z) = \frac{V_M^1(0)}{Z_1} e^{-\gamma_1 z} + \frac{V_M^2(0)}{Z_2} e^{-\gamma_2 z} \quad \text{(A.9.b)}$$

where the unknown constants (at $z = 0$) are determined by the boundary conditions, i.e., by driving and receiving circuitry.

In the nonchiral case (i.e., $U_M = J_M = 0$), (A.7) and (A.8) learn that $Z = Z_1 = -Z_2$, and $\gamma = \gamma_1 = -\gamma_2$. In that case (A.6) reduces then to the well-known transmission line equation:

$$-\frac{d}{dz}\begin{pmatrix} V_M(z) \\ I_M(z) \end{pmatrix} = \begin{pmatrix} 0 & \gamma Z \\ \frac{\gamma}{Z} & 0 \end{pmatrix} \begin{pmatrix} V_M(z) \\ I_M(z) \end{pmatrix} \quad \text{(A.10)}$$

and (A.7) reduces to

$$V_M(z) = V_M^1(0)e^{-\gamma z} + V_M^2(0)e^{+\gamma z} \quad \text{(A.11.a)}$$

$$I_M(z) = \frac{1}{Z}[V_M^1(0)e^{-\gamma z} - V_M^2(0)e^{+\gamma z}] \quad \text{(A.11.b)}$$

In the case of a single waveguide ($N = 1$) this modal description is identical to the circuit description. If $N > 1$ we use a matrix representation. First we group all relevant modal quantities in N by 1 column matrices ($\mathbf{V_M}$, $\mathbf{I_M}$, $\bar{\mathbf{E}}_t^{\mathbf{M}}$, $\bar{\mathbf{H}}_t^{\mathbf{M}}$, $\bar{\mathbf{E}}_l^{\mathbf{M}}$, $\bar{\mathbf{H}}_l^{\mathbf{M}}$, ...) and in N by N diagonal matrices ($\mathbf{Z_M}$, $\mathbf{Y_M}$, $\mathbf{U_M}$, $\mathbf{J_M}$, ...). Then we transform the modal description into a circuit description using (3) and (5). The extended telegrapher's matrix equations now become

$$-\frac{d}{dz}\begin{pmatrix} \mathbf{V}_c(z) \\ \mathbf{I}_c(z) \end{pmatrix} = \begin{pmatrix} \mathbf{U}_{cir} & \mathbf{Z}_{cir} \\ \mathbf{Y}_{cir} & \mathbf{J}_{cir} \end{pmatrix} \begin{pmatrix} \mathbf{V}_c(z) \\ \mathbf{I}_c(z) \end{pmatrix} \quad \text{(A.12)}$$

where

$$\mathbf{U}_{cir} = \mathbf{M_V U_M M_V^{-1}} \quad \text{(A.13.a)}$$

$$\mathbf{Z}_{cir} = \mathbf{M_V Z_M M_I^{-1}} \quad \text{(A.13.b)}$$

$$\mathbf{Y}_{cir} = \mathbf{M_I Y_M M_V^{-1}} \quad \text{(A.13.c)}$$

$$\mathbf{J}_{cir} = \mathbf{M_I J_M M_I^{-1}} \quad \text{(A.13.d)}$$

Acknowledgment

This work was supported by a grant to Tom Dhaene from the IWONL (Instituut tot Aanmoediging van het Wetenschappelijk Onderzoek in de Nijverheid en de Landbouw). Frank Olyslager is Research Associate and Daniël De Zutter is Research Director of the National Fund for Scientific Research of Belgium.

References

1. K.D. Marx, "Propagation Modes, Equivalent Circuits and Characteristic Terminations for Multiconductor Transmission Lines with Inhomogeneous Dielectrics," *IEEE Trans. Microwave Theory Tech.*, Vol. 21, No. 7, pp. 450–457, 1973.
2. I.V. Lindell, "On the Quasi-TEM Modes in Inhomogeneous Multiconductor Transmission Lines," *IEEE Trans. Microwave Theory Tech.*, Vol. 29, No. 8, pp. 812–817, 1981.
3. J.R. Brews, "Transmission Line Models for Lossy Waveguide Interconnections in VLSI," *IEEE Trans. Electron Devices*, Vol. 33, No. 9, pp. 1356–1365, 1986.
4. J.R. Brews, "Characteristic Impedance of Microstrip Lines," *IEEE Trans. Microwave Theory Tech.*, Vol. 35, No. 1, pp. 30–34, 1987.
5. R.H. Jansen, "Unified User-Oriented Computation of Shielded, Covered and Open Planar Microwave and Millimeter-Wave Transmission-Line Characteristics," *Microwaves, Optics, Acoustics*, Vol. 3, pp. 14–22, 1979.
6. L. Wiemer and R.H. Jansen, "Reciprocity Related Definition of Strip Characteristic Impedance for Multiconductor Hybrid-Mode Transmission Lines," *Microwave Optical Technol. Lett.*, Vol. 1, pp. 22–25, 1986.
7. N. Faché and D. De Zutter, "New High-Frequency Circuit Model for Coupled Lossless and Lossy Waveguide Structures," *IEEE Trans. Microwave Theory Tech.*, Vol. 38, No. 3, pp. 252–259, 1990.
8. T. Dhaene and D. De Zutter, "CAD-Oriented General Circuit Description of Uniform Coupled Lossy Dispersive Waveguide Structures," *IEEE Trans. Microwave Theory Tech.* Special Issue on Process-Oriented Microwave CAD and Modeling, Vol. 40, No. 7, pp. 1545–1554, 1992.
9. T. Dhaene, S. Criel, and D. De Zutter, "Analysis and Modeling of Coupled Dispersive Interconnection Lines," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-40, No. 11, pp. 2103–2105, 1992.
10. N. Faché, F. Olyslager, and D. De Zutter, *Electromagnetic and Circuit Modelling of Multiconductor Transmission Lines*, Clarendon Press: Oxford, England, 1993.
11. M. Geshiro, S. Yagi, and S. Sawa, "Analysis of Slotlines and Microstrip Lines on Anisotropic Substrates," *IEEE Trans. Microwave Theory Tech.*, Vol. 39, No. 1, pp. 64–69, 1991.
12. T. Dhaene, F. Olyslager, D. De Zutter, and P. Lagasse, "Transmission Line Modeling of Coupled Lossy Open Dispersive Hybrid Waveguide Structures in Isotropic and Anisotropic Media," *Int. Symp. Signals, Systems and Electronics* (URSI), Paris, pp. 427–429, 1992.
13. R. Griffith and M.S. Nakhla, "Mixed Frequency/Time Domain Analysis of Nonlinear Circuits," *IEEE Trans. Computer Aided Design*, Vol. 11, No. 8, pp. 1032–1043, 1992.
14. T. Dhaene, L. Martens, and D. De Zutter, "Transient Simulation of Arbitrary Nonuniform Interconnection Structures Characterised by Scattering Parameters," *IEEE Trans. Circuits Systems*, Special Issue on Simulation, Modeling, and Electrical Design of High-Speed and High-Density Interconnects, Vol. CAS-39, No. 11, pp. 928–937, 1992.

15. F. Olyslager, D. De Zutter, and K. Blomme, "Rigorous Analysis of the Propagation Characteristics of General Lossless and Lossy Multiconductor Transmission Lines in Multilayered Media," *IEEE Trans. Microwave Theory Tech.*, Vol. 41, No. 1, pp. 79–88, 1993.
16. A. Lakhtakia, V.K. Varadan, and V.V. Varadan, *Time Harmonic Fields in Chiral Media*, Lecture Notes in Physics, Springer-Verlag: Berlin, 1989.
17. I.V. Lindell, *Methods for Electromagnetic Field Analysis*, Clarendon Press: Oxford, England, 1992.

Tom Dhaene was born in Deinze, Belgium, on June 25, 1966. He received the degree in electrical engineering from the University of Ghent, Belgium, in 1989. He is currently working toward the Ph.D. degree in electrical engineering at the Laboratory of Electromagnetism and Acoustics (LEA) of the same university. His research focuses on all aspects of circuit modeling and circuit simulation of high-frequency and high-speed interconnections.

Frank Olyslager was born in Wilrijk, Belgium, on November 24, 1966. He received a degree in electrical engineering from the University of Ghent in July 1989. He obtained his Ph.D. at the same university in February 1993 with a thesis titled "Electromagnetic Modelling of Electric and Dielectric Waveguides in Layered Media." At present he works as research assistant of the National Fund for Scientific Research of Belgium (NFWO) at the Laboratorty of Electromagnetism and Acoustics. His research concerns the electromagnetic modeling of high-frequency electrical and optical interconnections and electromagnetic properties of bianisotropic materials.

Daniël De Zutter was born in Eeklo, Belgium, on November 8, 1953. He received a degree in electrical engineering from the University of Ghent in July 1976. From September 1976 to September 1984 he was a research and teaching assistant in the Laboratory of Electromagnetism and Acoustics (LEA) at the same university. In October 1981 he obtained a Ph.D. degree there and in the spring of 1984 he completed a thesis leading to a degree equivalent to the French Aggrégation or the German Habilitation. He is now a professor at Ghent University and research director at the National Science Foundation of Belgium. Most of his earlier scientific work dealt with the electrodynamics of moving media, with emphasis on the Doppler effect and Lorentz forces. His research now focuses on all aspects of circuit and electromagnetic modeling of high-speed and high-frequency interconnections. In 1990 he was elected as a member of the Electromagnetics Society.