GENERALIZED ITERATIVE FREQUENCY DOMAIN DECONVOLUTION TECHNIQUE

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Abstract

The deconvolution problem is mathematically classified as an ill-posed problem. In this paper, we present a new, fast, robust and accurate iterative frequency domain deconvolution technique which tackles this ill-posed problem.

Deconvolution

Consider a linear time invariant system with an impulse response h(t) (figure 1).

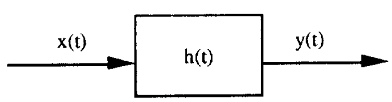

Figure 1 : Linear time invariant system

x(t) and y(t) are the input and output response respectively. Let $x_m(t)$ and $y_m(t)$ be the measured input and output response. Averaging may be used to increase the signal/noise ratio of the measurements. H(f), X(f), Y(f), $X_m(f)$ and $Y_m(f)$ are the corresponding frequency domain representations. Due to the inevitable measurement errors, the transfer function H(f) cannot be found exactly starting from $X_m(f)$ and $Y_m(f)$. We define the optimal transfer function $H_{opt}(f)$ as:

$$H_{opt}(f) = Y_m(f)\, F(f) / X_m(f) = H_m(f)\, F(f) \qquad (1)$$

where F(f) is a regularisation filter introduced to reduce the deconvolution noise. This filter must compensate all kinds of measurement errors. The choice of this filter is subjective.

We formulate a new general cost function K_{tot} which maximizes the smoothness of the deconvolved results, and minimizes the deconvolution errors and the unwanted peaks (caused by a division by nearly zero) over the frequency range $[0\text{-}f_{MAX}]$ which is useful for the envisaged application:

$$K_{tot} = K_{error} + \gamma\, K_{smooth} + \lambda\, K_{peak} \qquad (2)$$

where:

$$K_{error} = \int_0^{f_{MAX}} |\, Y_m(f) - H_{opt}(f)\; X_m(f)\, |^2\, df \qquad (3)$$

$$K_{smooth} = \int_0^{f_{MAX}} |\, (j\,\omega)^p\; H_{opt}(f)\, |^2\, df \qquad (4)$$

$$K_{peak} = \int_0^{f_{MAX}} |\, H_{opt}(f)\, |^2\, df \qquad (5)$$

p is a positive even number. γ and λ are real positive weighing factors.

The new generalized deconvolution filter F(f) is found by minimizing the cost function K_{tot} (2) with respect to F(f):

$$F(f,\gamma,\lambda) = \frac{|X_m(f)|^2}{|X_m(f)|^2 + \gamma\,\omega^{2p} + \lambda} \qquad (6)$$

The roll-off of this filter is mainly determined by the parameter p. Our new frequency domain deconvolution filter (6) combines the advantages of the "Guillaume-Nahman filter" ($\lambda = 0$) [1] and the "optimal compensation filter" ($\gamma = 0$) [2]. The parameter γ (smoothing factor) ensures an efficient reduction of the high frequency noise components, while the parameter λ (peak reduction factor) ensures a selective reduction of the deconvolution noise.

The optimal filter parameters γ and λ are calculated in the frequency domain. No time-consuming inverse Fourier transformations must be calculated as is the case in [1] and [2]. We look for a compromise between noise reduction and signal integrity. The noise reduction and the filter errors are controlled in different frequency ranges. This can be seen as an extended "Bennia-Riad criterium" [3] for two filter parameters. We partition the complete frequency range $[0 - f_{MAX}]$ of the transfer function in several regions i (i = 1, 2, ...) based on the information content in each interval. For example, in the case of an ideal low pass filter (figure 2) we have a pass band, a transition band, and a stop band [3].

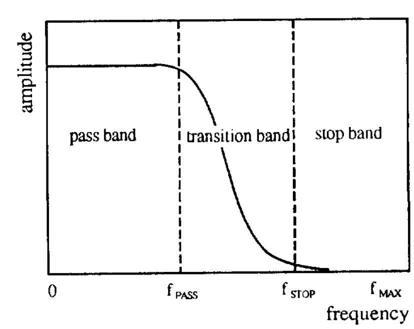

Figure 2: Low pass filter partitioned in three frequency regions

The pass band $[0 - f_{PASS}]$ contains most of the relevant frequency domain information (large values of $|X(f)|$). The noise level in the pass band is very low. The transition band $[f_{PASS} - f_{STOP}]$ still contains a

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certain amount of signal information, but the noise level is no more negligible. The stop band $[f_{STOP} - f_{MAX}]$ mainly contains noise, while the useful information content is negligible (small values of $|X(f)|$).

To select the optimal filter parameters, we calculate the normalized standard deviation of the magnitude of the transfer function as a function of the filter parameters in all different frequency regions i (i = 1, 2, ...). These so-called "noise factors" $NOISE_i$ are defined as:

$$NOISE_i(\gamma,\lambda) = RMS_i \{ \mid H_m(\omega) [F(f,\gamma,\lambda) - F(f,0,0)] \mid \} \quad (7)$$

The noise factors are normalized to their maximal value. If the filter parameters are too low, we have no effective noise reduction ($NOISE_i \approx 0$, for all i). On the other hand, if they are too large, the signal distortion is too high ($NOISE_i \approx 1$, for all i). We want to filter the noise as much as possible, but at the same time we want to limit the unwanted filter distortion.

For a low pass filter e.g., we first choose the initial filter parameters γ_{init} and λ_{init}:

$$\gamma_{init} = \frac{0.02 \ |X_{min}|^2}{(2\pi f_{PASS})^{2p}} \quad (8)$$

and:

$$\lambda_{init} = 0.02 \ |X_{min}|^2 \quad (9)$$

where $|X_{min}|^2$ is equal to the minimal value of $|X(f)|^2$ in the pass band $[0 - f_{PASS}]$. These initial parameters limit the maximal possible distortion in the pass band. Normally, they give already very good deconvolution results. Then, we vary the parameters γ and λ over some orders of magnitude and we monitor the normalized noise factors. The noise factors of the different regions can be vizualized with 3D- graphs or contour plots. The normalized noise factors must be very low in the pass band (region 1) and very high in the stop band (region 3). The "optimal" filter parameters γ_{opt} and λ_{opt} are chosen in such a way that: $NOISE_1 < 0.05$, $NOISE_2 \approx 0.6$ and $NOISE_3 > 0.95$. The choice of parameters remains subjective.

Example

In this example, we apply the new deconvolution technique to calculate the time domain reflection coefficient $s_{11}(t)$ (ref. imp. = 50 Ω) of a double step-in-width microstrip line. The cross section and the top view of this nonuniform microstrip are depicted in figure 3.

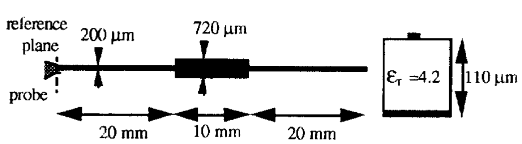

Figure 3: Double step-in-width microstrip line

The right hand side of the device under test (DUT) is open, while the left hand side of the DUT is connected with a HP 54121T reflectometer (bandwidth: 18.5 GHz) via high-quality 50 Ω coaxial cables and a coplanar high-frequency probe (Cascade Microtech PPH-100-150). The 50 Ω impedance and the short-circuit on the Cascade Microtech "Impedance Standard Substrate" (ISS) are used to calibrate the reflectometer [4]. In figure 4, the TDR-pictures $v_{load}(t)$ and $v_{short}(t)$ of the reference standards are shown. Waveform averaging is used (128 times) to improve the signal/noise ratio. The measured reflectogram of the nonuniform DUT $v_{dut}(t)$ and the reference load $v_{load}(t)$ are shown in figure 5.

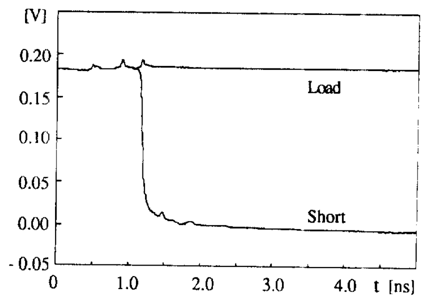

Figure 4: TDR-pictures of the 50 Ω and 0 Ω reference standards

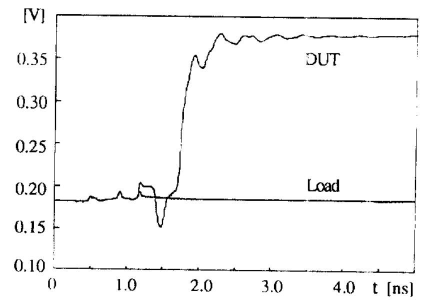

Figure 5: TDR-pictures of the 50 Ω standard and the DUT

All time domain data are transformed to the frequency domain. The frequency domain reflection coefficient $S_{11}(f)$ (ref. imp. = 50 Ω) is defined as:

$$S_{11} = -\frac{V_{dut} - V_{load}}{V_{short} - V_{load}} \quad (10)$$

The parameter p of the deconvolution filter (6) is chosen to be 4. The complete frequency range is partitioned into three frequency regions (pass band: 0-18.5 GHz, transition band: 18.5-24 GHz, stop band: 24-110 GHz). The maximal frequency f_{MAX} (=110 GHz) is determined by the sampling rate of the reflectometer and by the FFT algorithm. The maximal frequency of the pass band f_{PASS} (=18.5 GHz) is determined by the bandwidth of the oscilloscope. In figure 6, the contour plots of the noise factors in the three frequency domain regions are shown as a function of the filter parameters. The filter parameters are normalized to their initial guess.

We have optimal filtering if $NOISE_1 < 0.05$, $NOISE_2 \approx 0.6$ and $NOISE_3 > 0.95$. In that case we have a significant noise reduction in the stop band, a limited noise reduction in the transition band, and nearly no distortion in the pass band. The optimal filter parameters are located in the grey regions of figure 6. They are chosen to be: $\gamma_{opt} = 0.64 \ \gamma_{init}$ and $\lambda_{opt} = 0.64 \ \lambda_{init}$. The frequency domain reflection coefficient $S_{11}(f)$ is shown in figure 7 when no filter is

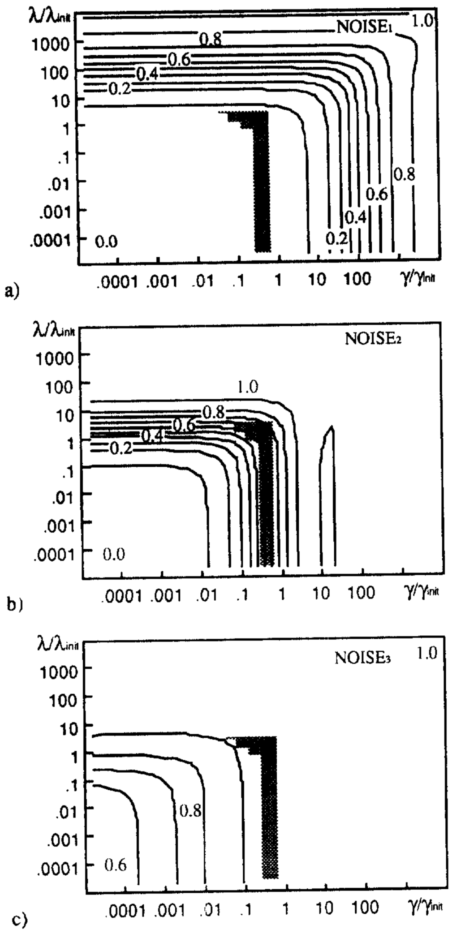

Figure 6: Noise factors as a function of the filter parameters in a) pass band, b) transition band, c) stop band

applied and when optimal filter parameters are chosen. The corresponding time domain impulse responses $s_{11}(t)$ are depicted in figure 8.

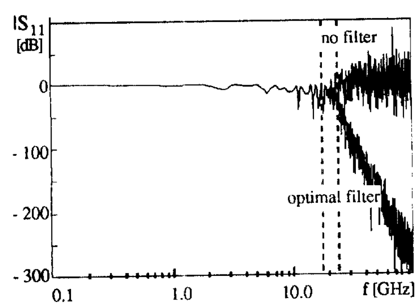

Figure 7: Frequency domain reflection coefficient $S_{11}(f)$

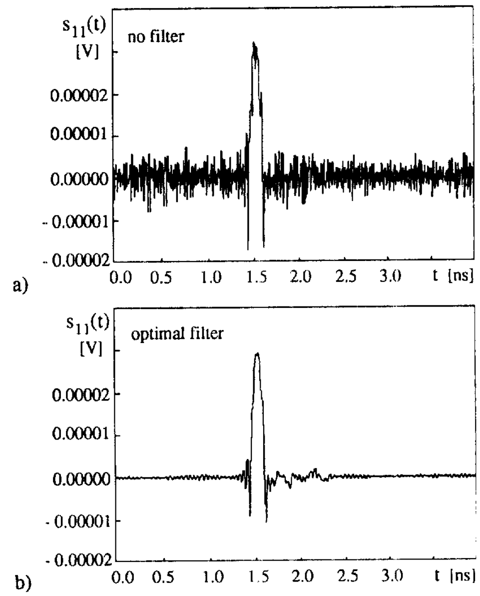

Figure 8: Time domain reflection coefficient $s_{11}(t)$, a) no filter, b) optimal filter parameters

Conclusions

In this paper, we developed a new advanced data processing technique for iterative frequency domain deconvolution. This deconvolution algorithm is much faster, more accurate, more robust and less sensitive to noise than traditional algorithms.

Acknowledgments

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References

[1] N. S. Nahman and M. E. Guillaume, "Deconvolution of Time Domain Waveforms in the Presence of Noise", National Bureau of Standards Technical Note 1047, Boulder (Colorado), October 1981.

[2] B. Parruck and S. M. Riad, "An Optimization Criterion for Iterative Deconvolution", IEEE Trans. on Instr. and Meas., Vol. IM-32, No. 1, pp. 137-140, March 1983.

[3] A. Bennia and S. M. Riad, "An Optimization Technique for Iterative Frequency-Domain Deconvolution", IEEE Trans. on Instr. and Meas., Vol. IM-39, No. 2, pp. 358-362, April 1990.

[4] Hewlett Packard, "Advanced TDR Techniques", HP-Application Note 62-3, May 1990.