TIME DOMAIN ANALYSIS OF UNIFORMLY COUPLED LOSSY TRANSMISSION LINES WITH ARBITRARY LOADS

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Abstract - Based on an extended scattering matrix (ESM) formulation, a new method is developed for time domain simulation of coupled dispersive lossy transmission lines with arbitrary linear or non-linear terminations. At each point in time, both sides of the transmission line structure are represented by an extended Thevenin equivalent. This general transmission line model is implemented in a SPICE-like simulation program.

Introduction

Transient analysis of coupled dispersive lossy transmission lines with non-linear loads is very important for the study and the design of high speed electronic circuits. The most common simulation approach of this general transient problem is based on a Green's function description. The basic procedure is always quite similar. First, the dispersive transmission line structures are evaluated in the frequency domain. Next, the frequency domain data are transformed into time domain Green's functions. Then, the problem is solved in a time-marching fashion: after each time step, relevant transmission line parameters are computed by convolving the Green's functions with previous voltage and current values.
The distinguishing feature between the different methods is the choice of the Green's functions. In the literature, Y-parameter and S-parameter descriptions have been used. The Y-parameter description introduces time-domain Green's functions of very long time duration, or artificial compensation networks [1]. On the other hand, the traditional scattering parameter descriptions of multiconductor lines (p.e. [2]-[3]) do not guarantee Green's functions of short time duration because generally speaking, the (resistive) reference impedances do not perfectly match.
In this contribution a new algorithm is presented for time domain simulation of lossy dispersive multiconductor transmission lines with arbitrary linear or non-linear terminations, based on an extended scattering matrix (ESM) formulation [4].

Equivalent Thevenin model

This section deals with the derivation of extended Thevenin models for uniform interconnection structures, first in the frequency domain, afterwards in the time domain.

Frequency domain analysis

Consider a set of N+1 transmission lines, where the $(N+1)^{th}$ line is the reference conductor. In the frequency domain, this linear passive structure is fully described by its extended scattering matrix [4]:

$$\begin{pmatrix} \mathbf{B}_1(\omega) \\ \mathbf{B}_2(\omega) \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{11}(\omega) & \mathbf{S}_{12}(\omega) \\ \mathbf{S}_{21}(\omega) & \mathbf{S}_{22}(\omega) \end{pmatrix} \begin{pmatrix} \mathbf{A}_1(\omega) \\ \mathbf{A}_2(\omega) \end{pmatrix} = \begin{pmatrix} \mathbf{0} & e^{-\Lambda_v(\omega) d} \\ e^{-\Lambda_v(\omega) d} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{A}_1(\omega) \\ \mathbf{A}_2(\omega) \end{pmatrix} \tag{1}$$

where $\mathbf{A}_i$ and $\mathbf{B}_i$ are the incident and reflected voltage wave vectors at port i (i = 1, 2). The complex coupled voltage propagation matrix is represented by Λ_v. Instead of using separate reference impedances for each port of the multiport, a global reference impedance matrix is chosen for each side of the transmission line structure. The reference matrix can have a resistive and a reactive part, and will be frequency dependent in the general case. This matrix is chosen to be equal to the characteristic impedance matrix $\mathbf{Z}_c$ of the structure. Note that $S_{11}=S_{22}$ and $S_{12}=S_{21}$ due to the symmetry and the reciprocity of the 2N-port.

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The voltages at both sides of the structure are related to the currents and the reflected voltage waves at the same side:

$$\mathbf{V}(0,\omega) = \mathbf{A_1}(\omega)+\mathbf{B_1}(\omega) = 2\,\mathbf{B_1}(\omega)+\mathbf{Z_c}(\omega)\,\mathbf{I}(0,\omega) \qquad (2a)$$

$$\mathbf{V}(d,\omega) = \mathbf{A_2}(\omega)+\mathbf{B_2}(\omega) = 2\,\mathbf{B_2}(\omega) - \mathbf{Z_c}(\omega)\,\mathbf{I}(d,\omega) \qquad (2b)$$

These equations describe the extended Thevenin circuit in the frequency domain.

Time domain analysis

The time domain formulation is found by using the inverse Fourier transformation (inverse FFT). The multiplications in the frequency domain become convolutions in the time domain. Note that the Green's functions have a relatively short duration due to the fact that the reference matrix is perfectly matched.
The simulation problem is handled in a time-stepping fashion. All equations are discretised before computer implementation. (2.a) and (2.b) transform to:

$$\mathbf{v}(0,q) = 2\sum_{p=1}^{q} \mathbf{s_{12}}(p)\,\mathbf{a_2}(q\text{-}p)\,\Delta t + \mathbf{Z_l}\,\mathbf{i}(0,q) + \sum_{p=1}^{q} \mathbf{z_c}(p)\,\mathbf{i}(0,q\text{-}p)\,\Delta t \qquad (3a)$$

$$= 2\,\mathbf{b_1}(q\text{-}1) + \mathbf{Z_l}\,\mathbf{i}(0,q) + \mathbf{v_z}(0,q\text{-}1)$$

$$\mathbf{v}(d,q) = 2\sum_{p=1}^{q} \mathbf{s_{12}}(p)\,\mathbf{a_1}(q\text{-}p)\,\Delta t - \mathbf{Z_l}\,\mathbf{i}(d,q) - \sum_{p=1}^{q} \mathbf{z_c}(p)\,\mathbf{i}(d,q\text{-}p)\,\Delta t \qquad (3b)$$

$$= 2\,\mathbf{b_2}(q\text{-}1) - \mathbf{Z_l}\,\mathbf{i}(d,q) + \mathbf{v_z}(d,q\text{-}1)$$

where q stands for the discretised time. $\mathbf{v_z}$ is a voltage source vector which depends on previous current waves at the same side of the transmission line structure. $\mathbf{b_1}$ and $\mathbf{b_2}$ can be seen as voltage source vectors depending on previous voltages waves. Based on the initial value theorem of transform analysis an instantaneous impedance matrix $\mathbf{Z_l}$ is defined.
At each point in time, both sides of the coupled transmission line network can be represented by an extended Thevenin equivalent (figure 1). An analogous analysis based on current wave vectors instead of voltage wave vectors leads to an extended Norton equivalent.

Practical considerations

All the time domain Green's functions ($s_{12}(t)$ and $z_{c_{pq}}(t)$) can be interpreted as impulse functions. Due to the fact that one has a perfect match, the time duration of these Green's functions will be short. Short impulse responses are important for the stability and accuracy of the simulation method. Shorter impulses facilitate the inverse transformation problem, increase the computational speed of the algorithm and decrease the memory requirements and the cumulated errors.

The time domain simulation algorithm is solved in a time-stepping fashion. At each point in time, both sides of the coupled dispersive lossy transmission line network are reduced to an equivalent network. In conjunction with the linear and/or non-linear loading circuits, the whole network can be handled with classical circuit techniques.

Numerical example

Consider the coupled transmission line network of figure 2. This example is also discussed in [1]. The active line is driven by a voltage source which generates a pulse of 6 ns with a rise and fall time of 1.5 ns. The left side of the passive line is terminated in a 75 Ω resistor. At the right side, both lines are terminated in a series combination of a 10 Ω resistor and a non-linear resistor. The relation between the current i through and the voltage v over the non-linear resistor is given by:

$$i = 10\left(\exp\left(\frac{v}{V_T}\right) - 1\right)\text{nA} \qquad (4)$$

where V_T=25 mV. The line parameter matrices are:

$$\mathbf{R} = \begin{bmatrix} 524 & 33.9 \\ 33.9 & 524 \end{bmatrix} \sqrt{f}\ \mu\Omega/\text{m}$$

$$\mathbf{L} = \begin{bmatrix} 309 & 21.7 \\ 21.7 & 309 \end{bmatrix} \text{nH/m} \qquad (5)$$

$$\mathbf{G} = \begin{bmatrix} 0.905 & -0.0118 \\ -0.0118 & 0.905 \end{bmatrix} f\ \text{pS/m}$$

$$\mathbf{C} = \begin{bmatrix} 144 & -6.4 \\ -6.4 & 144 \end{bmatrix} \text{pF/m}$$

The line-resistance matrix varies with the square root of the frequency f, due to the skin-effect losses, whereas the line-conductance matrix is proportional to the frequency. Figure 3.a shows the voltages at beginning and end of the driven line, and figure 3.b shows the forward and backward crosstalk on the sense line. The full lines represent the voltages at the left side of the network, while the dashed lines gives the results at the right side.

The non-linear elements were linearised with an iteration scheme based on the Newton-Raphson algorithm. The simulation time step was chosen to be 40 ps. The highest frequency involved in the computation of $\mathbf{s}_{12}$ and $\mathbf{z}_c$ was 62.5 GHz and 12.5 GHz respectively. The line transit time of the even mode is 3.376 ns, and 3.290 ns for the odd mode. The results are in good agreement with [1].

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Figures

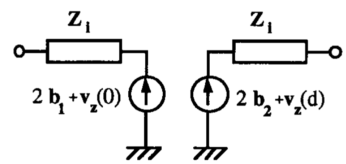

fig. 1 : Extended Thevenin equivalent.

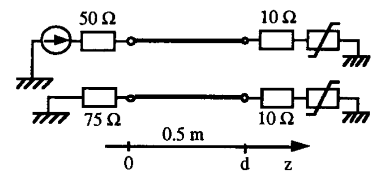

fig. 2: Coupled structure with non-linear loads [1].

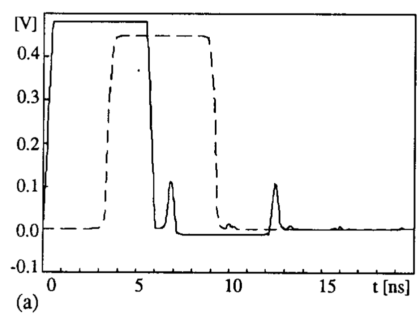

fig. 3.a : Voltages on driven line.

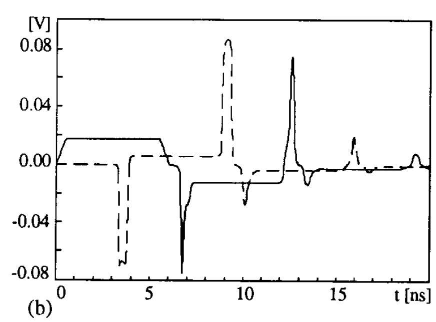

fig. 3.b : Voltages on sense line.