Multipoint Model Order Reduction for Systems with Delays

Elizabeth Rita Samuel*, Dirk Deschrijver*, Francesco Ferranti#, Luc Knockaert*, Tom Dhaene*

* iMinds, Gaston Crommenlaan 8 Bus 201, B-9050, Gent, Belgium, email: lizita3@gmail.com, {dirk.deschrijver, luc.knockaert, tom.dhaene}@ ugent.be

Vrije Universiteit Brussels, Pleinlaan 2, B-1050, Brussels, Belgium, email: francesco.ferranti@vub.ac.be

***Abstract* — An adaptive frequency sampling algorithm is proposed in this paper to automate the generation of reduced order models for systems with delays which can be represented as frequency dependent state-space matrices. Reflective exploration technique is used to obtain an optimum number of frequency samples for which the reduced state-space matrices per frequency is computed using a common projection matrix and is then interpolated to obtain the frequency response. The algorithm is illustrated using a numerical example.**

***Index Terms* — Model order reduction, reflective exploration, time delay systems.**

I. INTRODUCTION

Three-dimensional field solvers are required for the understanding of many engineering aspects like interconnect delay estimation, signal integrity analysis and electromagnetic compatibility applications [1]. The three-dimensional field solvers are very accurate but is at the cost of very large system of equations which are expensive to solve. To overcome this, model order reduction (MOR) techniques has been developed to speed up the electromagnetic (EM) simulations.

MOR reduces the overall complexity while retaining the important featuers of the original system. For quasi-static EM analysis, several robust MOR technques have been proposed [2]-[4] and also multipoint Krylov-based MOR for accurate response over a frequency range of interest [5]. Some techniques has also been proposed for the MOR of full-wave systems [6]-[7].

System with delays belongs to the class of functional differential equations and in Laplace domain contain elements with exponential factors $e^{-s\tau}$ where τ corresponds to the delay present in the circuit. It leads to complex algebraic systems of equations with frequency dependent state-space in the frequency domain. In this paper, a full-wave MOR is proposed for systems with delays using multipoint expansion technique. The expansion points are adaptively selected using reflective exploration technique [8] and the reduced state-space matrices per expansion point is computed using a common projection matrix [9]. Then the frequency dependent state space matrices are interpolated over the frequency range of interest to obtain the frequency response.

II. MODEL ORDER REDUCTION

Consider a time-delay system (TDSs) of degree n with p port and n_τ delays as:

$$\boldsymbol{E_0}\,\dot{\boldsymbol{x}}(t) + \sum_{j=1}^{n_\tau} \boldsymbol{E_j}\dot{\boldsymbol{x}}(t\text{-}\tau) = \boldsymbol{A_0}(\tau)\boldsymbol{x}(t) + \sum_{j=1}^{n_\tau} \boldsymbol{A_j}\boldsymbol{x}(t\text{-}\tau) + \boldsymbol{Bu}(t);$$

$$\boldsymbol{y}(t) = \boldsymbol{Cx}(t) \tag{1}$$

This can be written in the Laplace form as:

$$s\boldsymbol{E}(s)\boldsymbol{X}(s) = \boldsymbol{A}(s)X(s) + \boldsymbol{BU}(s)\ ; \quad \boldsymbol{Y}(s) = \boldsymbol{CX}(s) \tag{2}$$

Where, $\boldsymbol{E}(s) = \boldsymbol{E_0} + \sum_{j=1}^{n_\tau} \boldsymbol{E_j} e^{-s\tau_j}$; $\boldsymbol{A}(s) = \boldsymbol{A_0} + \sum_{j=1}^{n_\tau} \boldsymbol{A_j} e^{-s\tau_j}$.

Now the system has frequency dependent state-space matrices. For computing the frequency dependent reduced order state-space matrices the projection matrices are generated for each frequency sample by considering the expansion point as equal to the corresponding frequency sample.

Let s0 be the adaptively chosen expansion point, then the transfer function can be written as:

$$\boldsymbol{H}(s) = \boldsymbol{C}\,(\boldsymbol{I}\text{-}(s\text{-}s_0)\boldsymbol{M})^{-1}\,\boldsymbol{R} \tag{3}$$

Where, $\boldsymbol{M} = -(s_0\boldsymbol{E}(s)\text{-}\boldsymbol{A}(s))^{-1}\,\boldsymbol{E}(s)$; $\boldsymbol{R} = (s_0\boldsymbol{E}(s)\text{-}\boldsymbol{A}(s))^{-1}\boldsymbol{B}$.

Then the *q-th* block Krylov subspace is given by

$$\mathbb{K}q(\boldsymbol{M},\boldsymbol{R}) = colspan[\ \boldsymbol{R}\ \boldsymbol{MR}\ \boldsymbol{M}^2\boldsymbol{R}\ \ldots\ \boldsymbol{M}^{(q-1)}\boldsymbol{R}] \tag{4}$$

From the Krylov subspace (4), the column orthogonal matrix, $\boldsymbol{Vq}$ can be generated per frequency sample. Then by congruence transformation the frequency dependent reduced state-space matrices are obtained.

For m expansion points that are adaptively choosen the corresponding column orthogonal matrices V_{qi} for i=1,2, ..., m are obtained. Then a common projection matrix $\boldsymbol{V_{comm}}$ (5) is computed for obtaining the reduced state-space matrices that are then interpolated to obtain the frequency response.

$$\boldsymbol{V_{comm}} = [\ \boldsymbol{V_{q1}}\ \boldsymbol{V_{q2}}\ \ldots\ \boldsymbol{V_{qm}}] \tag{5}$$

III. PROPOSED TECHNIQUE

The goal of the proposed technique is to adaptively sample the frequency range using reflective exploration [8], for multipoint MOR of a system described using frequency dependent state-space matrices (2). Then using a common projection matrix [9], the reduced frequency dependent state-space matrices are obtained, which are then interpolated over the frequency range of interest to obtain the frequency response.

A. Reflective Exploration (RE)

The technique of selecting samples and modeling in an adaptive way is reffered to as RE. It is effective when the generation of the model is very expensive using an EM simulator. A reflective function is required for the sample selection and the proposed technique uses the root mean square (RMS) error for K_s number of frequency samples as defined

$$Err_{adp}^{I} = \sqrt{\frac{\sum_{sk=1}^{K_s}\sum_{i=1}^{p}\sum_{j=1}^{p}\left|H_{I,(ij)}(s) - H_{I-1,(ij)}(s)\right|^2}{p^2 K_s}} \qquad (6)$$

The algorithm has two loops: an adaptive mdeling loop and an adaptive sampling loop.

For the adaptive modeling loop, the exploration starts with two expansion points selected at [ω_{min} , ω_{max}] of the frequency range of interest. The reduced order per sample is considered to be equal to the number of ports p. The reduced model per sample is computed using a common projection matrix as in (5) and is interpolated over K_s number of frequency samples to obtain the frequency response. Then in the next iteration the reduced order is increased by p, and then again the response is computed over the same number of frequency samples. The model is iterated by increasing the reduced order per sample by p.

When the difference between the RMS error of the I^{th} and $(I\text{-}1)^{th}$ iteration is less than 10% then the adaptive sampling loop starts. In this loop the model in the I^{th} iteration ($\boldsymbol{H_I}$) is compared with the actual frequency response ($\boldsymbol{H_{act}}$) over the same number of frequency samples K_s, by computing the standard 2-norm per frequency.

$$\text{Err}_{sk} = \|\boldsymbol{H_{act}}(sk)\text{-}\boldsymbol{H_I}(sk)\| \; ; \; k=1,2, ..., K_s \qquad (7)$$

The frequency at which the Err_{sk} is maximum, a new frequency sample is considered. Then the iteration enters again the modeling loop and is continued till all the error per frequency Err_{sk} is below a threshold.

B. Compact Common Projection Matrix

After obtaining the projection matrices and the frequency dependent state-space matrices form the adaptively chosen sample points it would be further possible to compact the reduced state-space matrices using the singular values of the common projection matrix $\boldsymbol{V_{comm}}$ (5). The economy sized SVD is computed for the $\boldsymbol{V_{comm}}$

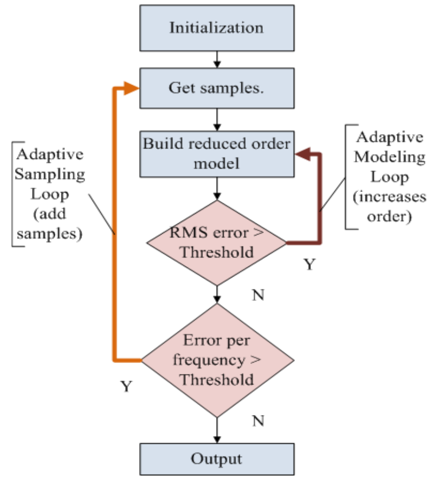

Fig. 1. Flowchart for Reflective exploration.

$$\boldsymbol{U\Sigma V^T} = \text{SVD}\; (\boldsymbol{V_{comm}}, \boldsymbol{0}) \qquad (8)$$

A common reduced order q_{comm} is defined based on the first q_{comm} significant singular values $\boldsymbol{\sigma}$ = diag ($\boldsymbol{\Sigma}$), by setting a threshold to the ratio of the singular values to the largest singular value. This threshold can be set with respect to the accuracy required for the model. Thus the common projection matrix for the samples chosen adaptively is

$$\boldsymbol{Q_{comm}}=\boldsymbol{U}(:,1:q_{comm}) \qquad (9)$$

Finally a congruence transformation is applied on the frequency dependent state-space that are generated for the adaptively chosen sample points, which are then interpolated to obtain the frequency response of the system.

VI. NUMERICAL RESULTS

A system consisting of two multiconductor transmission lines and two RLC networks has been considered as detailed in [10] over a frequency range of [1 kHz – 10 GHz]. The order of the original model is 1414 with 4 ports.

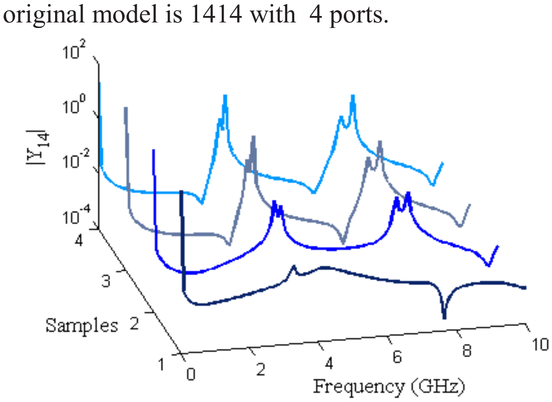

Fig. 2. Reduced order model obtained during the addition of new sample.

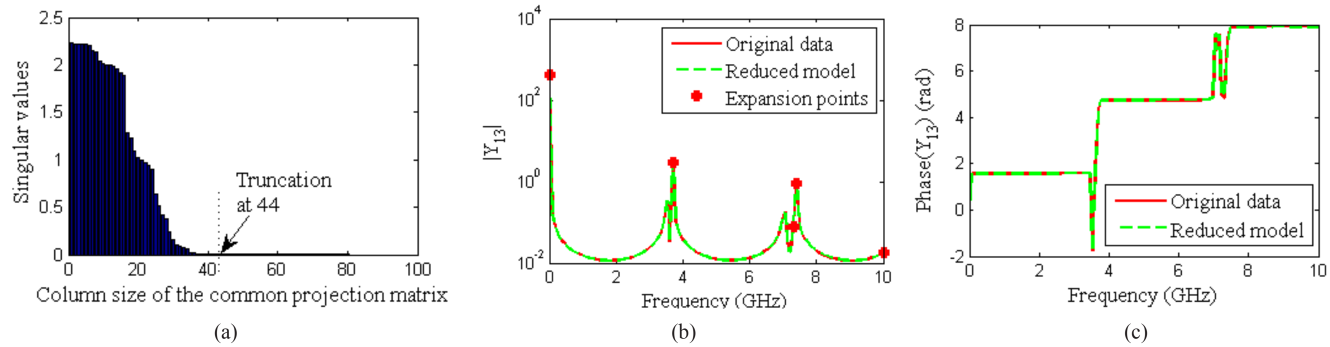

Fig. 3. (a) Singular values of the common projection matrix, (b) magnitude of Y_{13} with the expansion points and (c) phase plot of Y_{13}.

The exploration starts with two samples at [ω_{min} , ω_{max}] with an initial reduced order of 4. The response is computed and then compared with an increased reduced order of 8.

A RMS error (6) of 15.3 is obtained between the two models, the reduced order is further incremented. Then when the differences between the RMS errors of the I^{th} and $I\text{-}1^{th}$ is less than 10%, the algorithm enters the adaptive sampling loop and identifies the new sample frequency as explained in Section III-A.

The process iterates till the required accuracy is achieved. In Fig. 2. the reduced model obtained after the addition of new expansion points is shown. After obtaining the projection matrices per sample frequency a common projection matrix is formed as in (5) and then is truncated based on the singular values as shown in Fig. 3. (a). Fig 3. (b) and Fig. 3. (c) plots the magnitude and phase plot of Y13 of the original model and reduced model of order 44. Considering 120 frequency samples the RMS error achieved is 0.046 with a computation time of 25.6 s. Thus the proposed technique was able to achieve the required accuracy with a compact reduced model efficiently.

VII. Conclusion

The paper proposes an adaptive frequency sampling algorithm using reflective exploration for the generation of frequency dependent reduced order state-space matrices. The reduced order state-space matrices are obtained using a compact common projection matrix which is truncated using its singular values and then interpolated to generate the frequency response.

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