# Fast Optimization of Microwave Filters using Surrogate-Based Optimization Methods

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Abstract — This paper investigates the use of surrogate-based optimization to optimize the behavioral response of broadband microwave filters. The proposed method makes use of an efficient infill criterion (called expected improvement) that sequentially samples the response of the device at wellchosen regions in the design space. Based on these data samples, successive global surrogate models are built that become increasingly accurate near the optimum solution. A microwave filter example confirms that this approach significantly accelerates device optimization when compared to standard gradient-based methods.

### 1 Introduction

The design and optimization of passive microwave filters often requires a massive amount of Electro-Magnetic (EM) simulations in order to find the optimum design space parameters. Although several EM simulators are available to compute the response of a device, it is well-known that such simulations are often computationally expensive and slow down the overall optimization process [1, 2].

In [3], it was shown how to generate a parametric macromodel that is globally accurate over the complete design space. Such a model can act as a fast surrogate or replacement model for the simulator, and thereby allows considerable speedups during the optimization process [4]. The model can also be used in design space exploration, sensitivity analysis etc., as an add-on. However, the calculation of the parametric macromodel is not a trivial task, since the computational cost to build and validate the high-fidelity macromodel may require a considerable amount of additional EM simulations.

The surrogate-based optimization (SBO) method proposed in this letter takes advantage of the fact that full coverage of the complete design space is not needed to find an optimum solution. Rather than computing a globally accurate macromodel of the parameterized frequency response as in [3], it is proposed to generate a macromodel of the cost

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function that is locally accurate in the regions of interest. The method exploits one of the most popular adaptive sampling concepts, namely, the Expected Improvement (EI) measure for optimization with the help of Kriging surrogate models [5, 6, 7].

The EI measure guides the sequential selection of data samples into regions of the design space where the optimum solution is most likely to be found. Once the algorithm discovers a sample that satisfies the requirements, the optimization is terminated and the final solution is returned. The efficiency of the optimization method is illustrated by applying it to a double folded stub filter [8].

## 2 Surrogate-Based Optimization (SBO)

The parameterized transfer matrix  $H(s, \vec{q})$  of a device can be simulated as a function of the frequency  $s = j\omega$  and a vector of design variables  $\vec{q}$ . The aim of the optimization process is to find values for the design variables  $\vec{g}$  such that the system response satisfies a set of design specifications. These specifications are reformulated into a unified cost function that must be minimized (Sect. 2.1). First, a limited set of simulations are performed such that the cost function is well sampled over the parameter space (Sect. 2.2), and a Kriging surrogate model is built (Sect. 2.3). The Expected Improvement (EI) infill criterion is then used to select additional data samples in a sequential way, and the Kriging surrogate model is updated (Sect. 2.4). The EI infill criterion effectively balances between enhancing the global accuracy of the surrogate model (exploration) and improving its accuracy near the current optimum (exploitation). As the algorithm continues, the model becomes increasingly accurate near the optimum (i.e. the minimum of the cost function). As soon as a satisfactory solution is found, the optimization is terminated and the best solution is returned.

# 2.1 Definition of the cost function

A typical EM design optimization problem consists of several constraints on the lower  $(R_{\rm L}^k)$  and upper  $(R_{\rm U}^k)$  bounds on the magnitude of a frequency response at several frequencies  $s_k$ , k = 1, 2, ..., K.

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Each specification can be formulated as an individual cost function  $f_k(\vec{g})$  that needs to be minimized

$$f_k(\vec{g}) = R_{\rm L}^k - |H(s_k, \vec{g})|$$
 or  $|H(s_k, \vec{g})| - R_{\rm U}^k$ . (1)

The cost functions (1) are then unified in a minimax sense to yield the overall cost function

$$f(\vec{g}) = \max_{k} (f_k(\vec{g})). \tag{2}$$

Note that a negative cost indicates that the corresponding specifications are satisfied, while a positive cost denotes that the specifications are violated. The goal of the optimization procedure is to choose the design parameters  $\vec{g}$  in such a way that the overall cost function  $f(\vec{g})$  (2) is minimized.

# 2.2 Simulation of initial data samples

First, a limited number of EM simulations are performed to obtain the response  $H(s, \vec{g})$  for wellchosen values of the design variables  $\vec{g}$ . To this end, an optimized Latin Hypercube Design was used because of its space-filling properties [9]. Secondly, the corresponding outputs of the cost function  $f(\vec{g})$ in (1) and (2) are computed to build a Kriging surrogate model.

# 2.3 Generation of a Kriging surrogate model

Kriging models are part of a broader class of approximation methods, called Gaussian Processes (GP), and have a particular importance in SBO. While traditional approximation methods predict only a single function value, GP methods can predict the uncertainty of a function value as the realization of a normally distributed random variable  $Y(\vec{g}) \sim N(\mu(\vec{g}), \sigma^2(\vec{g}))$  where  $\mu(\vec{g})$  denotes the mean or predicted value  $(\mu(\vec{g}) \approx f(\vec{g}))$  and  $\sigma^2(\vec{g})$  denotes the prediction variance. This property is exploited by the infill criterion to guide the sequential sampling, as shown in the next section. The details of Kriging models are well documented in the literature [10, 6, 11].

#### 2.4 Expected Improvement infill criteria

Once a Kriging model is built, the EI measure determines the optimum location of the next infill point at which a simulation has to be carried out. First, the EI quantifies the amount of improvement that is expected to occur as compared to the optimum value obtained so far. The EI is computed by considering every possible improvement over the current best optimum value  $f_{\min}$ , multiplied by the



Figure 1: EI of an unknown function f(g) at g = 0.5. A Gaussian probability density function (PDF) is shown at g = 0.5 and the first moment of the shaded area represents the value of the EI [7].

associated likelihood. If  $\phi(\cdot)$  denotes the probability density function of a random variable, then the EI can be written in integral form as [5]

$$E[I(\vec{g})] = \int_{-\infty}^{f_{min}} I(\vec{g}) \cdot \phi(Y(\vec{g})) dY \qquad (3)$$

where the improvement  $I(\vec{g})$  of  $Y(\vec{g})$  over  $f_{\min}$  is defined as

$$I(\vec{g}) = \max(f_{min} - Y(\vec{g}), 0).$$
(4)

This function (3) can be expressed in a closed-form [7], and it is then optimized over the design space using the DIviding RECTangles (DIRECT) algorithm [12]. It automatically balances exploration and exploitation [5, 6]. For convenience, a graphical illustration of the EI concept is provided in Fig. 1. Note that the EI function (3) corresponds to the first moment of the shared area in Fig. 1. The additional samples generated this way are used to update the Kriging model, and the process is repeated until a satisfactory solution is found.

### 3 Numerical example: Double folded stub (DFS) microwave filter

The example deals with the optimization of a DFS microwave filter, where the spacing S between the stubs and the length L of the stubs are chosen as the design variables in addition to frequency (see Fig. 2). The substrate for the filter is chosen with relative permittivity  $\epsilon_r = 9.9$  and a thickness of 0.127 mm, and the ranges of the parameters are reported in Table 1. The design specifications of this band-stop filter are given in terms of the scattering parameters, similarly to [13],

 $|S_{21}| \ge -3 \,\mathrm{dB}$  for freq  $\le 9 \,\mathrm{GHz}$ , freq  $\ge 17 \,\mathrm{GHz}$ ,  $|S_{21}| \le -30 \,\mathrm{dB}$  for  $12 \,\mathrm{GHz} \le \mathrm{freq} \le 14 \,\mathrm{GHz}$ . (5)

Table 1: Design parameters of the DFS band-stop filter

Parameter	Min	Max
Frequency $(freq)$	$5~\mathrm{GHz}$	$20~\mathrm{GHz}$
Spacing $(S)$	0.1 mm	0.25  mm
Length $(L)$	2.0  mm	3.0  mm

The scattering element  $S_{21}(s, S, L)$  is computed using ADS Momentum [14], and the frequency samples are used to evaluate the cost function  $f(\vec{g})$  with  $\vec{g} = (S, L)$ , as in (1) and (2).

Figs. 3 and 4 show the response of the cost function  $f(\vec{g})$  and its contour plot respectively, where the samples selected by the infill-criterion are marked as black dots. It is found that the samples are indeed clustered near the optimum, while maintaining a good overall coverage of the design space. In practice, the algorithm is terminated as soon as a satisfactory solution  $(S^*, L^*)$  is found, in this case after simulating only 12 samples. The corresponding frequency response is visualized in Fig. 5, and it is straightforward to verify that the design specifications in (5) are indeed satisfied.

Table 2 makes a comparison with two other approaches that are reported in literature. The first one is the gradient-based minimax optimization routine as implemented in ADS Momentum. The second one involves the calculation of a parametric macromodel whose sensitivity information is exploited to speed-up the optimization process [3]. Both methods require an initial starting point  $(S^0, L^0)$  for the optimization, and are also able to find a satisfactory solution. Nevertheless, it turns out that the surrogate-based optimization method is able to find a solution with the smallest computation times. Another key advantage of this approach is that the simulated frequency samples can be stored and re-used to speed-up future optimization that involve different cost functions.



Figure 2: Layout of Double Folded Stub Filter



Figure 3: Cost function over the design space along with the sampled locations.



Figure 4: A contour plot of the cost function shown in Fig. 3.



Figure 5: Magnitude of  $S_{21}$  at the solution using proposed optimization.

Table 2: Overview different optimization routines

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Method		$  (S^0, L^0)  $	$  (S^*, L^*)  $	Time	$ f(g^*) $
		[mm]	[mm]	[sec]	[dB]
ADS		(0.1500,	(0.2303,		
Momentum		2.6364)	2.1580)	5115	0.00
Para-	Case	(0.1500,	(0.2408,		
metric	Ι	2.6364)	2.1802)	3734	-1.67
macro-	Case	(0.1500,	(0.2408,		
model	II	2.6364)	2.1802)	3716	-1.67
SBO					
optimization			(0.2499,		
(proposed		N/A	2.1708)	394	-1.63
approach)					

# 4 Conclusions

Surrogate-Based Optimization using the concept of Expected Improvement (EI) of microwave filters has been presented in this paper. This paper compares the SBO method with the approach of using parametric sensitivity macromodels, focusing on optimization and subsequently finding suitable values for the design parameters. A comparison has been made in terms of the number of EM simulations required and the computational time before an optimum is found.

### References

- A. Antonini, D. Deschrijver, and T. Dhaene, "Broadband rational macromodeling based on the adaptive frequency sampling algorithm and the partial element equivalent circuit method," *IEEE Transactions on Electromagnetic Compatibility*, vol. 50, no. 1, pp. 128 – 137, Feb. 2008.
- [2] D. Deschrijver and T. Dhaene, "Fast passivity enforcement of S -parameter macromodels by pole perturbation," *IEEE Transactions on Microwave Theory and Techniques*, vol. 57, no. 3, pp. 620–626, March 2009.
- [3] K. Chemmangat, F. Ferranti, T. Dhaene, and L. Knockaert, "Gradient-based optimization using parametric sensitivity macromodels," *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, 2012. [Online]. Available: http://dx.doi.org/10.1002/jnm.839
- [4] T. Dhaene and D. Deschrijver, "Generalised vector fitting algorithm for macromodelling of passive electronic components," *Electronics*

*Letters*, vol. 41, no. 6, pp. 299 – 300, March 2005.

- [5] D. R. Jones, "A taxonomy of global optimization methods based on response surfaces," *Journal of Global Optimization*, vol. 21, pp. 345–383, 2001.
- [6] A. I. Forrester and A. J. Keane, "Recent advances in surrogate-based optimization," *Progress in Aerospace Sciences*, vol. 45, no. 1-3, pp. 50 – 79, 2009.
- [7] I. Couckuyt, F. Declercq, T. Dhaene, H. Rogier, and L. Knockaert, "Surrogate-based infill optimization applied to electromagnetic problems," *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 20, no. 5, pp. 492–501, 2010.
- [8] D. Gorissen, I. Couckuyt, P. Demeester, T. Dhaene, and K. Crombecq, "A surrogate modeling and adaptive sampling toolbox for computer based design," *Journal Machine Learning Research*, vol. 11, pp. 2051–2055, August 2010.
- [9] V. Joseph and Y. Hung, "Orthogonal-maximin latin hypercube designs," *Statistica Sinica*, vol. 18, pp. 171–186, 2008.
- [10] G. Matheron, "Principles of geostatistics," *Economic Geology*, vol. 58, no. 8, pp. 1246– 1266, 1963.
- [11] I. Couckuyt, S. Koziel, and T. Dhaene, "Kriging, co-kriging and space mapping for microwave circuit modeling," in 2011 41st EuropeanMicrowave Conference (EuMC), Oct. 2011, pp. 444 –447.
- [12] D. R. Jones, C. D. Perttunen, and B. E. Stuckman, "Lipschitzian optimization without the Lipschitz constant," *Journal of Optimization Theory and Applications*, vol. 79, pp. 157–181, 1993.
- [13] J. Ureel and D. De Zutter, "Gradient-based minimax optimization of planar microstrip structures with the use of electromagnetic simulations," *International Journal of Microwave* and Millimeter-Wave Computer-Aided Engineering, vol. 7, no. 1, pp. 29–36, Jan. 1997.
- [14] "Agilent EEsof COMMS EDA, ADS Momentum Software. Agilent Technol. Inc., Santa Rosa, CA, 2009."