Robust Macromodeling of Frequency Responses with Outliers [*]

Dirk Deschrijver, Luc Knockaert, Tom Dhaene
Dept. of Information Technology, Ghent University - IBBT,
Sint Pietersnieuwstraat 41, 9000 Ghent, Belgium
email : {dirk.deschrijver, luc.knockaert, tom.dhaene}@intec.ugent.be

Abstract

This paper introduces a robust Vector Fitting algorithm for macromodeling of measured or simulated frequency responses with outliers. The use of a new frequency-dependent weighting scheme ensures that the complex fitting error is minimized in the L_1 sense, rather than the L_2 sense. Numerical results indicate that this approach leads to more accurate results.

1 Introduction

Robust macromodeling techniques are of paramount importance for efficient time domain and frequency domain simulation of passive linear microwave systems and devices. Vector Fitting (VF) has proved to be a fast and reliable method that calculates high-order transfer function from measured or simulated frequency responses. It has been adopted in many societies of applied engineering, including power systems and microwave systems. A recent survey of the methodology is found in [1].

The standard Vector Fitting algorithm computes a macromodel by minimizing a weighted iterative cost function in the L_2 sense [2][3]. However, in real-life situations, it is possible that outliers in the data strongly degrade the quality of the L_2 fitting model. Outliers are values in the frequency response that deviate strongly from the other values, and they can be caused by various measurement or instrumentation errors [4].

This paper discusses a modified Vector Fitting algorithm that minimizes the L_1 norm of the complex fitting error instead of the L_2 norm [5, 6]. It gives a more elaborate explanation of the derivations in [7] and presents a new example which demonstrates that the approach is more robust towards outliers [8].

2 L_1-norm Macromodeling Algorithm

Based on a discrete set of S-parameter data samples $\{s_k, H(s_k)\}_{k=0}^{K}$, VF computes a rational macromodel with numerator $N^t(s)$ and denominator $D^t(s)$ in an iterative way ($t = 1, ..., T$) by successively solving least squares problems

$$\arg \min \sum_{k=0}^{K} \left|W^t(s_k)\right|^2 \left|(\sigma H)^t(s_k) - \sigma^t(s_k)H(s_k)\right|^2 \quad (1)$$

As shown in [1] and [9], both $(\sigma H)^t(s)$ and $\sigma^t(s)$ belong to a linear span of $p = 1, ..., P$ rational basis functions $\Psi_p(s, a^{t-1})$ that are based on the previously identified set of poles a^{t-1}. In the first iteration step ($t = 1$) these initial poles a^0 are chosen by the standard heuristical scheme to ensure a good numerical conditioning [2]. The basis functions Ψ are typically chosen as

partial fractions or orthonormal rational functions [10]

$$(\sigma H)^t(s) = \frac{N^t(s)}{D^{t-1}(s)} = c_0^t + \sum_{p=1}^{P} c_p^t \Psi_p(s, a^{t-1}) \quad (2)$$

$$\sigma^t(s) = \frac{D^t(s)}{D^{t-1}(s)} = 1 + \sum_{p=1}^{P} \tilde{c}_p^t \Psi_p(s, a^{t-1}) \quad (3)$$

Note that the cost function (1) also contains a user-defined weighting function $W^t(s)$. This weighting function should not be confused with the Sanathanan-Koerner weighting, in this paper denoted as $w^t(s)$, which occurs implicitly by pole relocation. In a standard application of Vector Fitting, the value of $W^t(s)$ is set to 1 by default. However, it is well known that an additional frequency-dependent weighting factor can provide a powerful way to control the accuracy of the macromodel.

In the following sections, it will be shown that the L_1 norm of the complex fitting error can be minimized by selecting the user-defined weighting factor $W^t(s)$ in (1) as follows

$$W^t(s) = \frac{\sqrt{|H(s) - H^{t-1}(s)|}}{H(s) - H^{t-1}(s)} \quad (4)$$

where $H^{t-1}(s) = N^{t-1}(s)/D^{t-1}(s)$ denotes the frequency response of the macromodel at previous iteration step $t - 1$.

3 Proof Outline

First, let's define the auxiliary function $f(s)$ as follows

$$f(s) = \frac{|D^t(s)H(s) - N^t(s)|^2}{|D^{t-1}(s)H(s) - N^{t-1}(s)|^2} \quad (5)$$

Applying the weighting factor $W^t(s)$ in (4) to the cost function (1) yields the following equivalent expression

$$\arg \min \sum_{k=0}^{K} \left|H(s_k) - H^{t-1}(s_k)\right| f(s_k) \quad (6)$$

Hence, upon convergence of the iterative scheme ($D^{t-1} \to D^t$ and $N^{t-1} \to N^t$), it follows that $f(s) \to 1$. Therefore, it is clear that cost function (6) effectively minimizes the L_1 norm of the complex fitting error $\left\|H(s) - H^{t-1}(s)\right\|_1$.

4 Analysis of the weighting function

In this section, it will be shown how the weighting function $W^t(s)$ in (4) was derived. The objective of the standard Vector Fitting procedure is to identify the model parameters in such a way that the L_2 norm of the complex error is minimized

$$\arg \min \sum_{k=0}^{K} \left|H(s_k) - \frac{N(s_k)}{D(s_k)}\right|^2 \quad (7)$$

To avoid non-linear optimization techniques, a linear set of equations is obtained by minimizing Levi's cost function [11]

$$\arg\min \sum_{k=0}^{K} |D(s_k)H(s_k) - N(s_k)|^2 \qquad (8)$$

This cost function (8) can easily be generalized by introducing a frequency-dependent weighting function, denoted as $Q(s)$

$$\arg\min \sum_{k=0}^{K} |Q(s_k)|^2 |D(s_k)H(s_k) - N(s_k)|^2 \qquad (9)$$

Note that (7) and (9) are equivalent if $Q(s)$ is set to $1/D(s)$. In order to minimize the L_1 norm of the complex fitting error, a different weighting function $Q(s)$ is chosen, such that

$$|Q(s)|^2 = \frac{|H(s) - N(s)/D(s)|}{|D(s)H(s) - N(s)|^2} \qquad (10)$$

Since $N(s)$ and $D(s)$ are not known in advance, they are replaced by the estimated numerator $N^{t-1}(s)$ and denominator $D^{t-1}(s)$ from the previous iteration step. Just like in the Sanathanan-Koerner iteration [12], inserting these values gives an estimate $Q^t(s)$ of the weight function $Q(s)$ that satisfies

$$
\begin{aligned}
|Q^t(s)|^2 &= \frac{\left|H(s) - N^{t-1}(s)/D^{t-1}(s)\right|}{\left|D^{t-1}(s)H(s) - N^{t-1}(s)\right|^2} \\
&= \frac{1}{|D^{t-1}(s)|^2} \frac{\left|H(s) - N^{t-1}(s)/D^{t-1}(s)\right|}{\left|H(s) - N^{t-1}(s)/D^{t-1}(s)\right|^2} \\
&= \frac{1}{|D^{t-1}(s)|^2} \frac{\left|H(s) - H^{t-1}(s)\right|}{\left|H(s) - H^{t-1}(s)\right|^2} \qquad (11)
\end{aligned}
$$

The use of weighting function $Q^t(s)$ leads to updated values of $N^t(s)$ and $D^t(s)$ in successive iteration steps ($t = 1, ..., T$)

$$\arg\min \sum_{k=0}^{K} |Q^t(s_k)|^2 \left|D^t(s_k)H(s_k) - N^t(s_k)\right|^2 \qquad (12)$$

Inserting (11) in (12) yields the following cost function

$$\arg\min \sum_{k=0}^{K} |W^t(s_k)|^2 \left| \frac{N^t(s_k)}{D^{t-1}(s_k)} - \frac{D^t(s_k)}{D^{t-1}(s_k)} H(s_k) \right|^2 \qquad (13)$$

Using the notations in (2) and (3), this is equivalent to

$$\arg\min \sum_{k=0}^{K} |W^t(s_k)|^2 \left|(\sigma H)^t(s_k) - \sigma^t(s_k)H(s_k)\right|^2 \qquad (14)$$

From (11), (12) and (13), $W^t(s)$ must satisfy the condition

$$|W^t(s)|^2 = \frac{\left|H(s) - H^{t-1}(s)\right|}{\left|H(s) - H^{t-1}(s)\right|^2} \qquad (15)$$

So, in order for (15) to hold, $W^t(s)$ is chosen as follows

$$W^t(s) = \frac{\sqrt{|H(s) - H^{t-1}(s)|}}{H(s) - H^{t-1}(s)} \qquad (16)$$

which corresponds to the user-defined weighting factor of Vector Fitting in (4). It is found that explicit weighting by $Q^t(s)$ in (12) is equivalent to explicit weighting by $W^t(s)$ in (14) + pole relocation (i.e. implicit weighting by $w^t(s) = 1/D^{t-1}(s)$) [1].
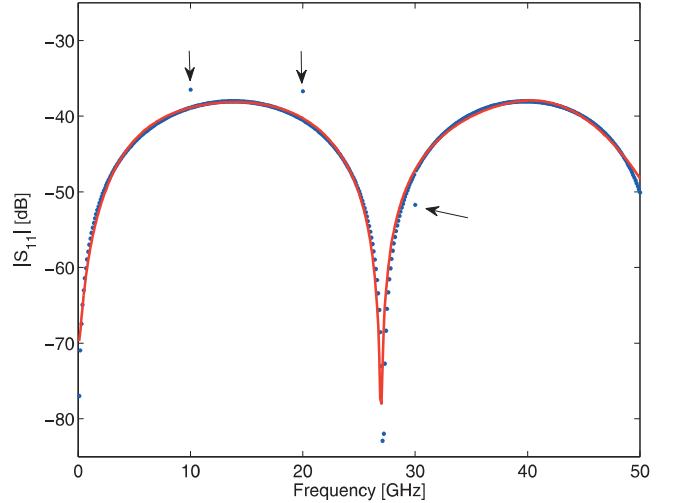

Figure 1: Data samples of reflection coefficient S_{11} (blue dots) versus magnitude response of the L_2 model (red line).
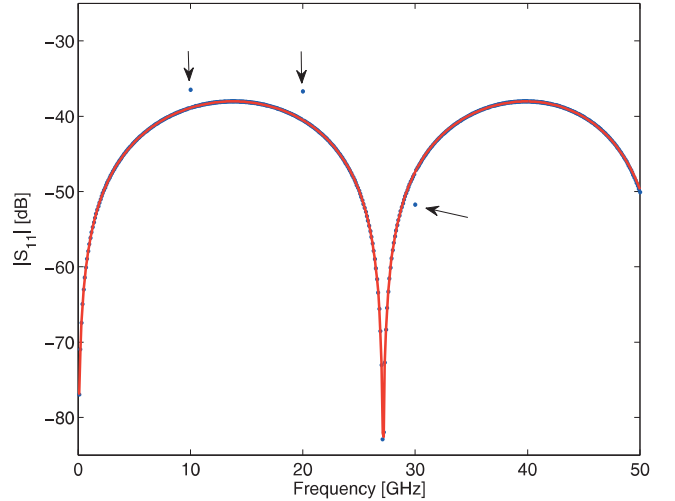

Figure 2: Data samples of reflection coefficient S_{11} (blue dots) versus magnitude response of the L_1 model (red line).

5   Example : Coplanar Waveguide

The reflection coefficient S_{11} of a 2-port coplanar waveguide is considered over the frequency range [0.1 GHz - 50 GHz]. Suppose that, due to inaccuracies in the data acquisition process, the S-parameter response contains three outlying data samples which are marked by black arrows in Figs. 1 to 4.

All the data samples are modeled by a rational 6-pole proper transfer function using the proposed VF methodology (L_1 norm) and the standard VF algorithm (L_2 norm). The magnitude of the data and the magnitude response of the model is shown in Figs. 1 and 2 respectively. Also the phase of the data is compared to the phase response of the model, as shown in Figs. 3 and 4. It is clear that the L_1 norm approximation yields an overall accurate result, and is not much affected by the presence of the outliers. On the other hand, the outliers lead to an undesired degradation of the model quality for the L_2 norm
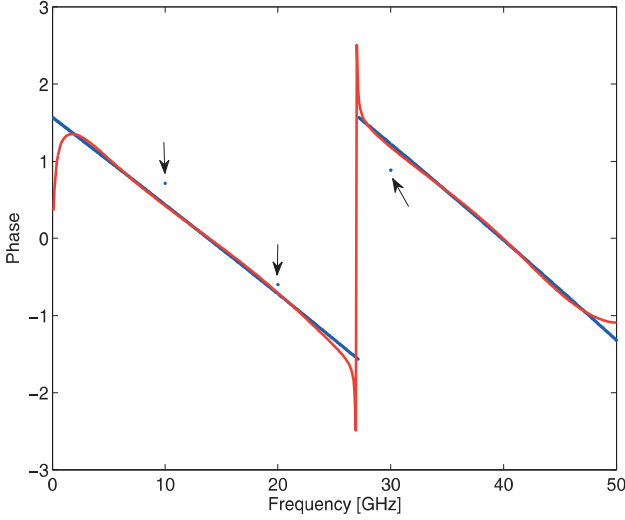
Figure 3: Data samples of reflection coefficient S_{11} (blue dots) versus phase response of the L_2 model (red line).
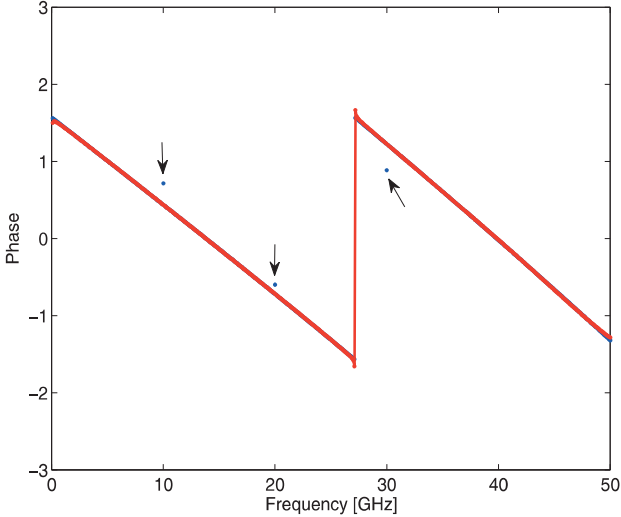

Figure 5: Absolute error L_1 model (green) and L_2 model (blue). The data samples contain 3 outliers, shown in Figs 1-4.

Figure 4: Data samples of reflection coefficient S_{11} (blue dots) versus phase response of the L_1 model (red line).

Figure 6: Magnitude of weighting function $W^t(s)$.

approximation. This observation is also confirmed by Fig. 5 where the absolute fitting error of both models is shown.

$$\text{Absolute fitting error} = |H(s) - H^{t-1}(s)| \qquad (17)$$

Fig. 6 visualizes the magnitude of the weighting function $W^t(s)$, defined in (16), as a function of the frequency. It is seen that three sharp spikes occur at the exact frequencies where the outliers are located. This indicates that the algorithm automatically detects the occurrence of outliers, and gives them a very small weight when compared to the other data samples. In the general case, this leads to a better accuracy than the classical VF algorithm, where $W^t(s)$ is constant for all frequencies.

As an additional test, the three outliers are manually removed from the data set and both modeling algorithms are applied to the remaining data samples. It is seen from Fig. 7 that the absolute error of both fitting models (L_1 and L_2) is comparable.
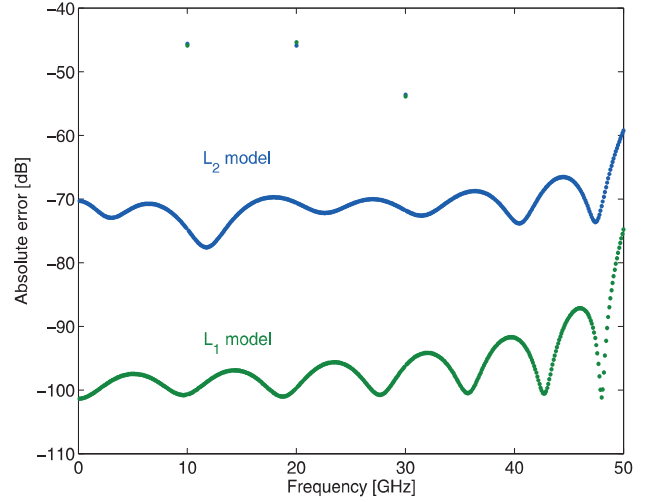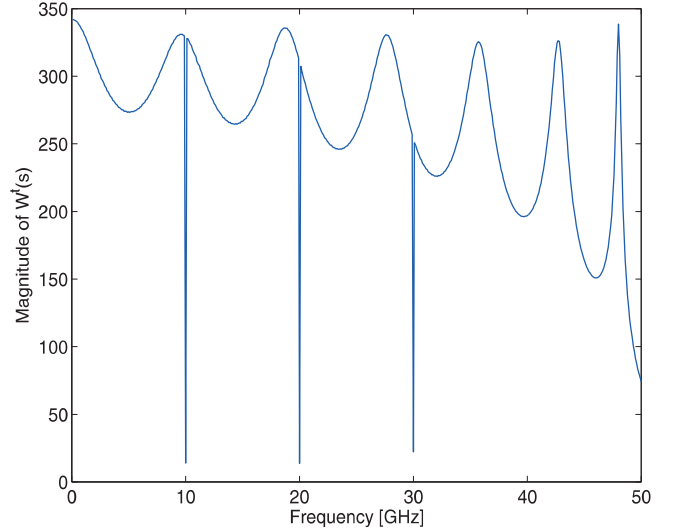
This result confirms that L_1 norm approximation is particularly useful in cases where the data is contaminated with outliers.

Conclusions

A modified Vector Fitting algorithm is proposed for L_1 norm identification of broadband macromodels from S-parameter data. The effectiveness of the algorithm is illustrated by applying it to a coplanar waveguide example, and the results are compared to the standard VF approach. It is found that the method is more robust when the frequency response contains outliers.

References

[1] D. Deschrijver, B. Gustavsen and T. Dhaene, "Advancements in Iterative Methods for Rational Approximation in the Frequency Domain", IEEE Transactions on Power Delivery, vol. 22, no. 3, pp. 1633-1642, 2007.

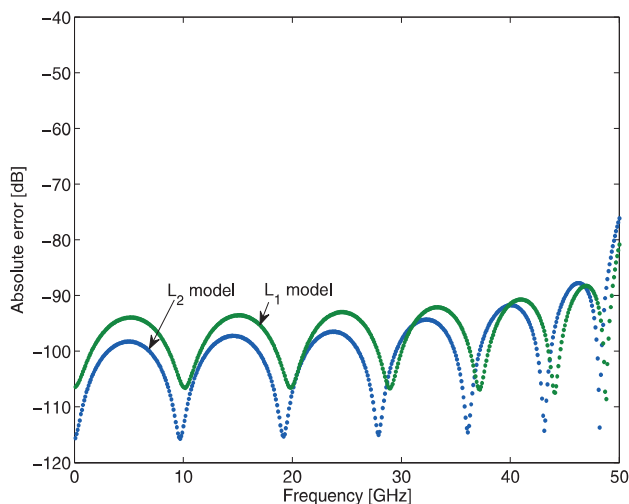[2] B. Gustavsen and A. Semlyen, "Rational Approximation of

Figure 7: Absolute error L_1 model (green) and L_2 model (blue). All the outliers are removed from the set of data samples.

Frequency Domain Responses by Vector Fitting", IEEE Transactions on Power Delivery, vol. 14, no. 3, pp. 1052-1061, 1999.

[3] W. Hendrickx, D. Deschrijver and T. Dhaene, "Some Remarks on the Vector Fitting Iteration", Progress in Industrial Mathematics at EMCI 2004, Mathematics in Industry, vol. 8, Springer-Verlag, pp. 134-138, 2006.

[4] R. Pintelon, J. Schoukens and Y. Rolain, "Frequency-Domain Approach to Continuous-Time System Identification: Some Practical Aspects", Advances in Industrial Control, pp. 215-248, 2008.

[5] C. Burrus, J. Barreto and I. Selesnick, "Iterative Reweighted Least-Squares Design of FIR Filters", IEEE Trans. on Signal Processing. vol. 42, no. 11, pp. 2926-2936, 1994.

[6] C.U. Lei, Y. Wang, Q. Chen, N. Wong, "On Vector Fitting Methods in Signal/Power Integrity Applications", in Proc. International Multiconference of Engineers and Computer Scientists, vol 2, pp. 1407-1412, 2010.

[7] D. Deschrijver, L. Knockaert and T. Dhaene, "Improving Robustness of Vector Fitting to Outliers in Data ", IEE Electronics Letters, vol. 46, no. 17, pp. 1200-1201, 2010.

[8] A. Kijko, "Seismologic Outliers: L_1 or Adaptive L_p Norm Application", Bulletin of the Seismologic Society of America, vol. 84, no. 2, pp. 473-477, 1994.

[9] D. Deschrijver and T. Dhaene, "A Note on the Multiplicity of Poles in the Vector Fitting Macromodeling Method", IEEE Transactions on Microwave Theory and Techniques, vol. 55, no. 4, pp. 736-741, 2007.

[10] D. Deschrijver, B. Haegeman and T. Dhaene, "Orthonormal Vector Fitting: A Robust Macromodeling Tool for Rational Approximation of Frequency Domain Responses",

IEEE Transactions on Advanced Packaging, vol. 30, no. 2, pp. 216-225, 2007.

[11] E.C. Levi, "Complex Curve Fitting", IRE Transactions on Automatic Control, vol. 4, pp. 37-44, 1959.

[12] C.K. Sanathanan and J. Koerner, "Transfer Function Synthesis as a Ratio of two Complex Polynomials", IEEE Transactions on Automatic Control, vol. 8, no. 1, pp. 56-58, 1963.