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Fast broadband modeling of frequency-domain responses by piecewise interpolation

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ABSTRACT

Broadband macromodeling of frequency domain responses by Vector Fitting can be computationally demanding in CPU time and memory. This paper presents a practical solution to reduce the computational cost that is involved with the modeling of high-order frequency responses. It applies a piecewise fitting strategy that makes use of a fast rational interpolation scheme to identify a representative set of data samples and an appropriate model order. This information is exploited by the Vector Fitting algorithm to extract the poles of the broadband macromodel in a reduced amount of time. The calculation of the residues is then solved as a linear approximation problem, and standard model reduction techniques can be applied as an optional step to remove pole redundancies in the model. It is shown by an example that substantial savings are obtained in terms of computation time and memory requirements, when compared to the standard fitting procedure.

1. Introduction

The accurate simulation of power system transients requires the usage of fast broadband macromodeling techniques. Such techniques are used for extracting frequency-dependent models of devices and subnetworks starting from measured or computed responses in the frequency domain. The use of rational macromodels leads to highly efficient transient simulations in the time domain, and the inclusion in electromagnetic transient programs (such as EMTP) is straightforward. Unfortunately, the calculation of accurate macromodels from highly resonant frequency responses can be time-consuming and memory demanding. High-order responses frequently occur when modeling power system subnetworks by a frequency-dependent network equivalent (FDNE).

Standard modeling approaches make use of Levi's polynomial-based fitting or an iterative weighted least-squares procedure to identify the coefficients of the macromodel [1]. Due to the structure of the associated least-squares matrix, it is known that such methods are prone to numerical ill-conditioning, especially for high-order cases. This makes the approach inadequate for the wide band modeling of network equivalents and transformers. A possible solution to overcome this difficulty is the partitioning of the frequency band into multiple sections along the imaginary axis. Each partition is modeled separately to identify the poles, and the residue matrices are calculated afterwards by fitting the overall response with the earlier identified poles. In [2], Min suggests to estimate the order of each partition by using a minimum eigenvalue tracking method, or by counting the number of resonance peaks. Inaccuracies in the overall macromodel are reduced by means of sub-band reordering, sub-band dilation and pole replacement. Noda [3] improves the accuracy and robustness of the fitting method by using an adaptive weighting scheme, column scaling and iteration step adjustment. The model order of each partition is estimated by a stepwise increase of the number of poles until some predefined accuracy level is reached.

Since the introduction of robust fitting techniques, such as Vector Fitting [4–7], most of the numerical inaccuracies can nowadays be avoided. Nevertheless, it is found that this method may become computationally inefficient if the modeling of frequency responses requires a very high order. The identification of the poles involves the solution of a system of equations using a QR decomposition, which has cubic complexity. Also, an eigenvalue problem needs to be solved which scales cubically with the size of the problem. Several iterations may be required to relocate the poles to an acceptable position, and this process needs to be repeated several times in order to find an appropriate model order. This makes the approach impractical for situations with a high order and many frequency samples. Solving the Normal Equations (NE) is a possible

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Table 1
Algorithm overview.

Step 1	Partition the frequency response into multiple partitions
Step 2	Subject each partition to adaptive sampling and order estimation using a fast rational interpolation procedure, until the desired accuracy is reached
Step 3	Subject each partition to pole extraction by ROVF based on the data sampling and model order that was found in Step 2
Step 4	Calculate the residues of the compound expression by fitting the overall response with the poles that are found in Step 3
Step 5	Remove redundant poles in the model (optional)

way to reduce computation time and memory consumption, as it makes the least-squares matrices more compact. However, wide-band responses may still require excessive computer resources. In addition, usage of normal equations may also result in an undesired loss of accuracy due to numerical ill-conditioning.

In order to overcome the above problem, a fast piecewise rational interpolation technique is combined with an adaptive sampling strategy. This approach selects a reduced set of data samples which characterizes the entire frequency response. At the same time, a good estimate for the model order is obtained. Based on this information, the Vector Fitting technique [4] with relaxation [5] and orthonormalization [6] (ROVF) is used to calculate the poles in a reduced amount of time [7]. Optionally, standard model reduction techniques can be applied to remove pole redundancies in the model.

2. Identification of poles

The approximation of a simulated frequency response $(s, H(s))$ by a rational macromodel $R(s)$ can be very time consuming, and may require a prohibitive amount of computer resources. In order to alleviate the computational burden, the frequency range of interest $[f_{min}, f_{max}]$ is divided into several neighboring partitions which are pair-wise disjoint. The width of each partition is divided in such a way that the total amount of resonant peaks is equally divided over the partitions. Each partition is treated as a separate frequency response, and it is subjected to a rational interpolation procedure (Section 2.1) that makes use of an adaptive sample selection and order estimation scheme (Section 2.2). Afterwards, the poles of each partition are extracted by ROVF (Section 2.3), and are used to calculate the residues of the compound response (Section 3). The approach is summarized in Table 1.

2.1. Fast rational interpolation using Thiele-type continued fractions

This section describes a fast rational interpolation technique that is based on Thiele-type continued fractions [8]. If a sparse set of $k+1$ data samples $(s,H(s))$ is given, then a rational interpolating function $R_k(s)$ can easily be obtained by forming a finitely truncated continued fraction of the following form:

$$R_k(s) = H(s_0) + \cfrac{s - s_0}{\phi(s_0, s_1) + \cfrac{s-s_1}{\phi(s_0,s_1,s_2) + \cfrac{\cdots}{\cdots + \cfrac{s-s_{k-1}}{\phi(s_0,\ldots,s_{k-1},s_k)}}}} \tag{1}$$

This function is compactly represented as

$$R_k(s) = H(s_0) + \sum_{v=1}^{k} \cfrac{s - s_{v-1}|}{|\phi(s_0, \ldots, s_{v-1}, s_v)} \tag{2}$$

The interpolant (1) is uniquely defined by the coefficients ϕ, which are called the *inverse* or *reciprocal* differences [9–12]. They form the rational counterpart of Newton's *divided* differences and they are computed recursively from the selected data samples in the following way

$$\phi(s_0, s_v) = \frac{s_0 - s_v}{H(s_0) - H(s_v)} \tag{3}$$

$$\phi(s_0, \ldots, s_{v-1}, s_v) = \frac{s_v - s_{v-1}}{\phi(s_0, \ldots, s_{v-2}, s_v) - \phi(s_0, \ldots, s_{v-2}, s_{v-1})}$$

Initially, the first data sample $(s_0, H(s_0))$ is interpolated by the constant function $R_0(s) = H(s_0)$. As a new data sample $(s_1, H(s_1))$ is selected, the continued fraction (1) can efficiently be updated by computing the corresponding inverse difference $\phi(s_0, s_1)$, and by adding a tail to the expansion, resulting in $R_1(s)$. Additional data samples can be interpolated in a similar way by expanding the fraction recursively. Once all data samples are selected and interpolated, the rational model can easily be evaluated at intermediate frequencies s by a bottom-up evaluation of (1). It is possible to reformulate the continued fraction as a ratio of two polynomials using three-term recurrence formulas. The theoretical details about this procedure are well-reported in numerical analysis textbooks, see [10]. It is noted that this interpolation scheme is fast, since it does not require any matrix inversion.

2.2. Sample selection and order estimation

It is known that the order of the rational function increases stepwise with the number of interpolated data samples, according to a staircase diagram [10]. To find a good estimate of the model order, it suffices to use a very basic adaptive sampling algorithm which selects a quasi-minimal set of key frequency samples that characterize the response of the partition. It is described as follows:

Initially, four data samples are selected which are equidistantly spread over the frequency range of interest. Based on these data samples, a rational interpolant (1) is calculated for each element of the transfer matrix, as described in Section 2.1. In successive iteration steps, the frequency response of the model is evaluated and compared to the reference data at intermediate frequencies. An additional data sample is selected at the frequency of largest mismatch, and the interpolant is expanded by adding a tail to (1). This process is repeated until the interpolant approximates the overall response of the partition sufficiently well, up to some predefined accuracy level.

Therefore, it is guaranteed that the model order is chosen sufficiently high to approximate the reference data, while the sparse sample distribution avoids overfitting of the model. Since the rational interpolant corresponding to each matrix element is based on the same number of samples, it is clear that the model order for each matrix element is the same.

2.3. Pole-identification by relaxed orthonormal vector fitting

Although it is possible to extract the poles of (1) from a three-term recurrence relation, it is found that this approach often generates very large or very small values from the partial denominators, thereby causing underflow or overflow of the floating-point representation [13]. Also, the model does not satisfy physical properties of a system, such as e.g. causality and stability of the poles.

To resolve this problem, the pole-identification step of ROVF is used to identify a set of stable poles in a robust way [4–7]. Based on the selected data samples and corresponding model order of (1), as computed in Section 2.2, it calculates a rational function approximation of the following form:

$$R(s) = \frac{(\sigma H)(s)}{\sigma(s)} = \frac{\sum_{p=1}^{P} c_p \Phi_p(s,a) + d + sh}{\tilde{c}_0 + \sum_{p=1}^{P} \tilde{c}_p \Phi_p(s,a)} \tag{4}$$

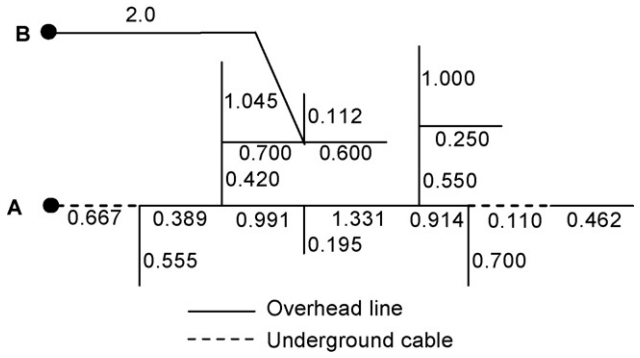

Fig. 1. Power system distribution system (lengths in km).

The basis functions $\Phi_p(s, a)$ are orthonormal rational functions which are based on a prescribed set of starting poles a [6]. These poles are chosen according to a heuristical scheme that was proposed in [4]. Based on the selected data samples and the corresponding model order that was determined in Section 2.2, the coefficients c_p, d, h and $\tilde{c}_p$ of the transfer function are found by iteratively minimizing the Sanathanan–Koerner cost function [1]. The relaxed non-triviality constraint in [5] is applied in order to improve the convergence properties of this iterative scheme. Once the final solution has been attained, the poles are found by forming the minimal state-space realization of $\sigma(s)$ and solving an eigenvalue problem. All details about this procedure are reported in [7].

3. Identification of residues

Once the procedure in Sections 2.1–2.3 is applied to each partition, all the poles and the selected data samples are gathered to compute a pole-residue model of the compound response [4]. One could apply partitioning to the identification of residues as well, but it was found that the entire frequency response should be considered at once to obtain a robust and reliable procedure.

4. Example: FDNE identification

As an example, frequency dependent network equivalent (FDNE) identification of a power distribution system is considered. The system has two 3-phase buses as terminals (A, B), and is shown in Fig. 1. The 6×6 admittance matrix $Y(s)$ is calculated with respect to these terminals in the frequency range 10 Hz–1 MHz. All lines and cables are modeled in the phase domain while taking into account the frequency dependent effects in conductors and ground. It was shown in [15] that representing this subnetwork by a macromodel (FDNE) can lead to much faster time domain simulations compared to a detailed representation by traveling wave models.

All timing results are calculated with a Pentium 4 laptop computer with a clock frequency of 2.66 GHz and 448 MB RAM memory on a Windows XP environment using MATLAB.

The proposed technique is used to calculate a broadband pole-residue model with stable poles only. To reduce the computational workload, the frequency range is divided in several, equally spaced partitions, which are modeled with a target root-mean-square (RMS) error of approximately 1E−4.

Fig. 2 shows the frequency response of all matrix elements (solid line), and the deviation of the calculated rational fitting (dashed line). In this case, the number of intervals is chosen to be 40 and an overall RMS fitting error of 4E−5 is obtained, which is clearly sufficient. Fig. 3 shows a zoom of Fig. 2, which shows that a good agreement is found between the fitting model and the reference data. In this figure, the selected data samples are also marked with crosses. It is seen that the interpolation approach results in a non-uniform sampling, which is more dense around the resonant frequencies.

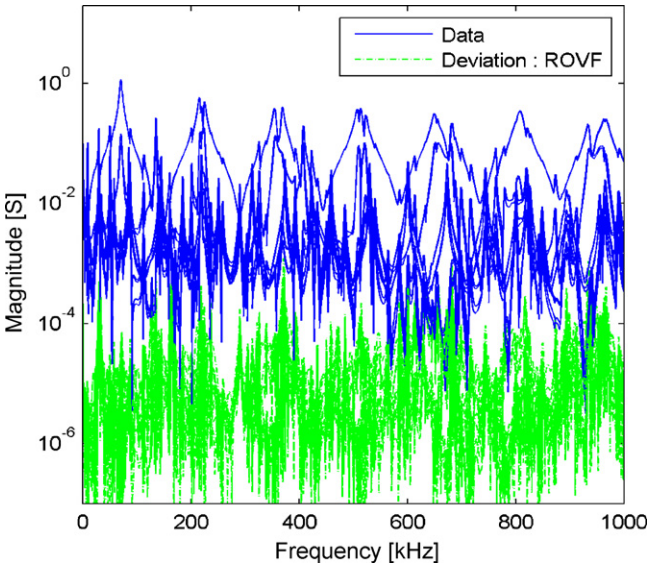

Fig. 2. Resulting rational approximation: 40 equally spaced partitions.

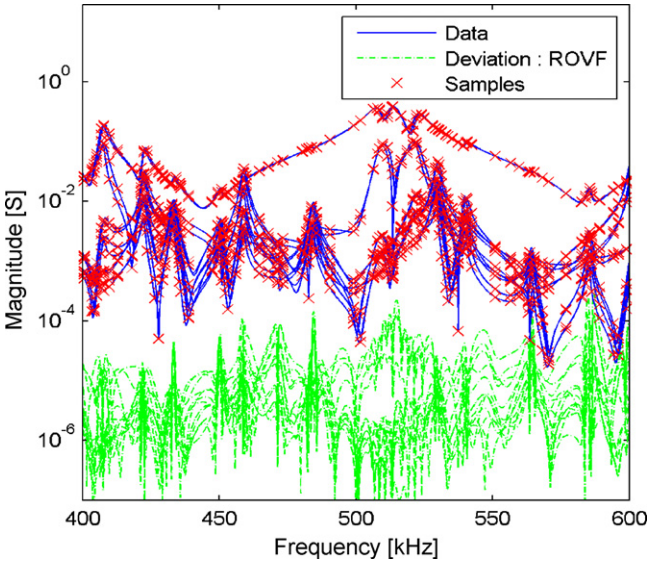

Fig. 3. Zoom of Fig. 2 over the frequency range 400–600 kHz.

Table 2 shows the overall accuracy and model order, for a varying number of partitions (from 5 to 60). Each sub-interval has been subjected to adaptive sampling and interpolation to reach a desired accuracy of RMS = 1E−4. It is observed that the overall RMS error of the broadband model corresponds quite well to the desired accuracy for a varying number of partitions (from 5 to 60). This confirms that the approach is robust, and that the accuracy of the model is fairly insensitive towards the choice of partitioning.

Table 2
RMS error and model order vs. partitioning.

#Part.	RMS	#Poles	#Part	RMS	#Poles
5	1.5E−4	394	35	2.6E−4	540
10	1.5E−4	416	40	4.4E−5	562
15	8.6E−5	448	45	6.5E−5	582
20	6.9E−5	478	50	4.3E−5	626
25	8.9E−5	498	55	4.0E−5	646
30	6.6E−5	532	60	3.2E−5	658

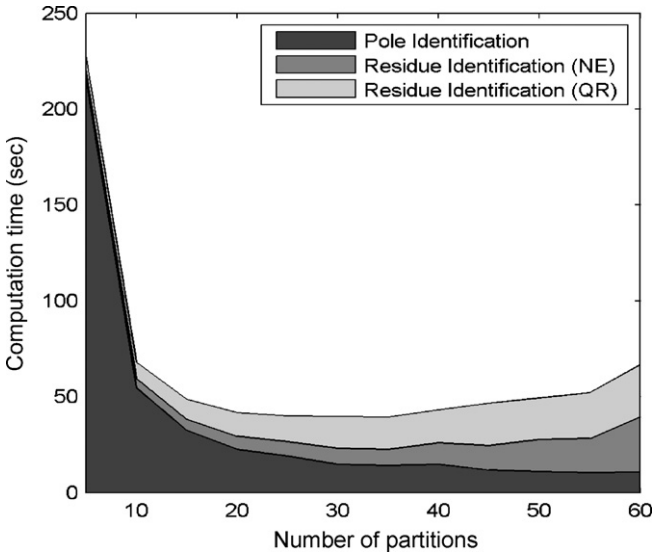

Fig. 4. Computation time vs. number of partitions.

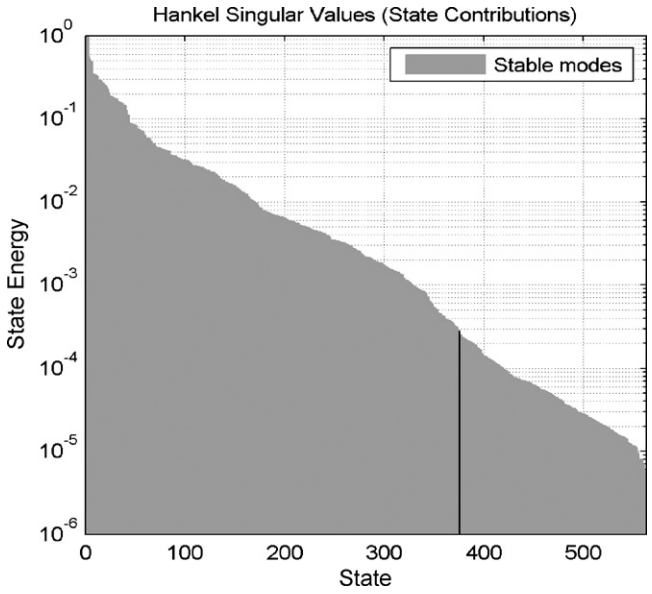

Fig. 6. Hankel singular values (40 partitions, reduced from 562 to 375 poles).

Fig. 4 shows the computation time of the proposed algorithm for a varying number of partitions. The black area indicates the computation time of the pole identification including the Thiele pre-processing step (Sections 2.1–2.3). The dark-grey area represents the computation time of the residue identification (Section 3) if the normal equations (NE) are solved, while the light-grey area denotes the *additional* computational cost if QR decomposition is used instead of NE.

It is clear that the computation time of the pole identification can be significantly reduced by using a larger amount of partitions. However, the number of poles increases with the number of partitions, and therefore the residue identification eventually becomes computationally more expensive. Nevertheless, Fig. 4 shows that the computational overhead of selecting *too many* partitions (>40) is fairly small as compared to the situation when *too few* partitions (<20) are chosen. For this example, it is found that 35 partitions is a good compromise. In this case, the overall macromodeling time corresponds to 39.42 s if the residues are calculated using a QR decomposition, or 22.63 s if the normal equations are solved.

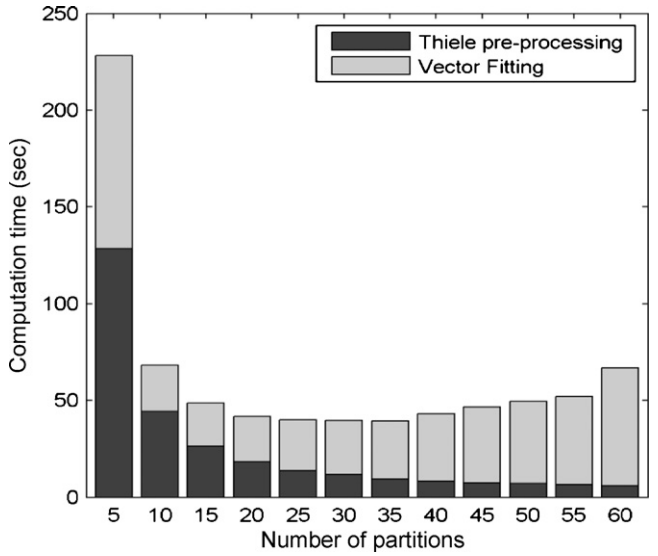

Fig. 5. Computation time of Thiele pre-processing step.

The black area in Fig. 5 shows the computation time of the Thiele pre-processing step, which is used to select a sparse sampling and appropriate model order (Sections 2.1 and 2.2). The grey area shows the computation time of the VF calculations (Sections 2.3 and 3). The sum of both areas in Fig. 5 equals the sum of the areas in Fig. 4 if the residues are calculated using QR decomposition. It shows that the computational cost of the pre-processing step is only a small fraction of the overall macromodeling time if the number of partitions is chosen sufficiently high.

It is noted that the amount of poles (or selected frequency samples) increases with the number of partitions. Therefore, the model may contain redundant poles which are not needed to have a good overall approximation, even though they are required to fit some partition of the overall response. This results from the fact that poles in the neighboring frequency ranges are neglected during the pole identification of each partition. To remove the redundancy in the model, standard Model Order Reduction methods can be applied [14]. Fig. 6 shows the Hankel singular values of the model, which measure the contribution of each state to the input–output behavior of the system. Small Hankel singular values indicate that some states can be discarded to simplify the model, using the 'balred' function in MATLAB. It is found that the RMS deviation between the 375-pole reduced order model and the reference data corresponds to 1.0E−4, which matches the desired accuracy. The additional computation time (48.91 s) is reasonably small when compared to the use of no partitioning.

Based on the results from Table 2, it is shown in Fig. 7 that the number of poles and the number of partitions are linearly correlated. By fitting a linear regression model to these values, it is possible to extrapolate the number of poles that would be needed if partitioning is not applied (#partitions = 1). It turns out that approximately 378 poles are required to fit the overall response. This corresponds closely to size of the reduced model, which indicates that nearly all redundant poles are removed.

It is clear that the computation time of the ROVF step would be much higher if the pre-processing step was not used. A direct application of the default ROVF routine has been considered to fit the compound response without partitioning or adaptive sampling. It was found that the modeling of the 36 matrix elements using 10,000 equidistant data samples is computationally infeasible, especially if no prior knowledge about the model order, and the required number of ROVF iterations is available.

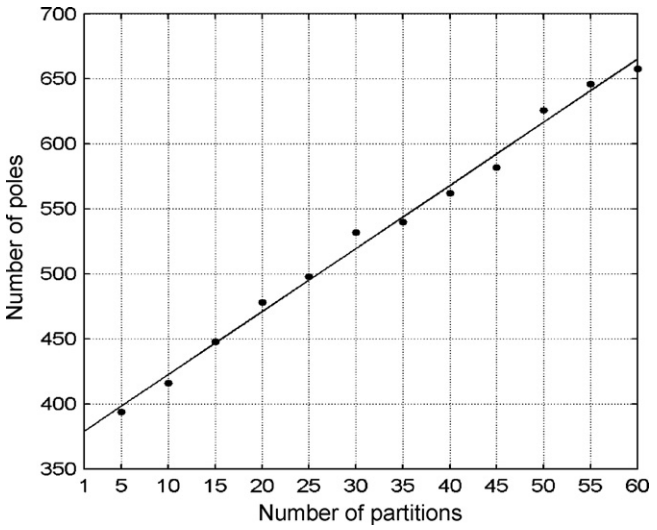

Fig. 7. Extrapolation of linear regression model.

5. Conclusions

Rational fitting of broadband frequency-domain responses using Relaxed Orthonormal Vector Fitting becomes computationally expensive and resource demanding if the bandwidth and complexity of the structure increases. Rational interpolation using Thiele-type continued fractions is used as a fast pre-processing step to calculate a set of representative data samples and to determine the model order in advance. This information is exploited by the ROVF algorithm to compute a physical pole-residue model in a significantly reduced amount of time. As an optional step, standard model reduction techniques can be applied to remove the redundant states in the macromodel.

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References

[1] C. Sanathanan, J. Koerner, Transfer function synthesis as a ratio of two complex polynomials, IEEE Trans. Automatic Control 8 (1) (1963) 56–58.
[2] S.H. Min, M. Swaminathan, Construction of broadband passive macromodels from frequency data for simulation of distributed interconnect networks, IEEE Trans. Electromagn. Compatibility 46 (4) (2004) 544–558.
[3] T. Noda, Identification of a multiphase network equivalent for electromagnetic transient calculations using partitioned frequency response, IEEE Trans. Power Deliv. 20 (2) (2005) 1134–1142.
[4] B. Gustavsen, A. Semlyen, Rational approximation of frequency domain responses by vector fitting, IEEE Trans. Power Deliv. 14 (3) (1999) 1052–1061.
[5] B. Gustavsen, Improving the pole relocating properties of vector fitting, IEEE Trans. Power Deliv. 21 (3) (2006) 1587–1592.
[6] D. Deschrijver, B. Haegeman, T. Dhaene, Orthonormal vector fitting: a robust macromodelling tool for rational approximation of frequency domain responses, IEEE Trans. Adv. Packaging 30 (2) (2007) 216–225.
[7] D. Deschrijver, B. Gustavsen, T. Dhaene, Advancements in iterative methods for rational approximation in the frequency domain, IEEE Trans. Power Deliv. 22 (3) (2007) 1633–1642.
[8] T.H. Thiele, Interpolationsrechnung, Teubner, Leipzig, 1909.
[9] F.B. Hildebrand, Introduction to Numerical Analysis, McGraw-Hill, New York, 1956.
[10] J. Stoer, R. Bulirsch, Introduction to Numerical Analysis, Springer-Verlag, New York, NY, 2002.
[11] J. Wallis, Opera Mathematica, vol. 1, Oxoniae e Theatro Shedoniano, 1695, p. 355. (Reprinted by Georg Olms Verlag, Hildesheim, Newyork, 1972).
[12] R. Lehmensiek, P. Meyer, An efficient adaptive frequency sampling algorithm for model-based parameter estimation as applied to aggressive space mapping, Microwave Opt. Technol. Lett. 24 (1) (1999) 71–78.
[13] W.H. Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery, Numerical Recipes in Fortran, 2nd ed., Cambridge University Press, 1986.
[14] A. Varga, Balancing-free square-root algorithm for computing singular perturbation approximations, in: Proceedings of 30th IEEE Conference on Decision and Control, UK, 1981, pp. 1062–1065.
[15] B. Gustavsen, O. Mo, Interfacing convolution based linear models to an electromagnetic transients program, in: Proceedings of International Conference on Power Systems Transients, France, 2007, p. 6.

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