Hybrid algorithm for compact and stable macromodelling of parameterised frequency responses

F. Ferranti, D. Deschrijver, L. Knockaert and T. Dhaene

A novel hybrid technique for the macromodelling of parameterised frequency responses is presented. A more compact model representation is proposed that guarantees stability of the poles by construction. It is shown that accurate parametric macromodels can be calculated using a reduced amount of model coefficients and data samples. The obtained macromodels can be used for efficient real-time design space exploration, design optimisation and sensitivity analysis.

Introduction: Real-time design space exploration, design optimisation and sensitivity analysis requires the development of accurate parametric approximation models [1], describing the dynamic behaviour of a system that is characterised by several design parameters in addition to time or frequency. In the past, it was common practice to store a dense set of simulated data samples in a multidimensional look-up table. Some local interpolation methods are then applied to extract the model response from neighbouring data points. Nowadays, the excessive disc storage requirements of this approach are avoided by computing compact parametric macromodels. Such methods have the ability to model the global behaviour of a system by a multivariate rational model using a limited amount of model coefficients [2]. This Letter shows that the number of model coefficients can be further reduced by combining the advantages of [2] with [3]. It is shown by an example that the hybrid technique is able to calculate more compact macromodels without compromising the accuracy of the results. It is found that the new method becomes less sensitive to the sample density and that the stability of the poles is preserved by construction. A comparison with previous results in [2] is reported.

Hybrid macromodelling algorithm: The proposed approach computes a stable parametric macromodel $R(s, \boldsymbol{g})$ from a multivariate set of data samples $\{(s, \boldsymbol{g})_k, H(s, \boldsymbol{g})_k\}_{k=1}^{K}$, where $s = j\omega$ is the complex frequency variable and $\boldsymbol{g} = \{g^{(n)}\}_{n=1}^{N}$ is a set of real design variables. It was proposed in [2] to represent the parametric macromodel as a partial fraction expansion with parameterised residues:

$$R(s, g^{(1)}, \ldots, g^{(N)}) = \sum_{p=1}^{P} \frac{c_p^{(N)}(g^{(1)}, \ldots, g^{(N)})}{s + a_p} \tag{1}$$

The poles $a = \{-a_p\}_{p=1}^{P}$ are found by fitting the frequency responses for all design parameter combinations using a common set of poles. A fast QR-based implementation of the relaxed vector fitting technique [4] is used to reduce the computational time and memory requirements of this step [5].

Then, the parameterised residues $c_p^{(N)}(g^{(1)}, \ldots, g^{(N)})$ are modelled by a modified version of the multivariate orthonormal vector fitting technique in [3]. It represents the residues as a multivariate rational function and computes the coefficients $c_{v1}, \ldots, v_N$ and $\tilde{c}_{v1}, \ldots, v_N$ by an iterative least-squares fitting procedure:

$$c_p^{(N)}(g^{(1)}, \ldots, g^{(N)}) = \frac{\sum_{v_1=0}^{V_1} \cdots \sum_{u_N=0}^{V_N} c_{v_1,\ldots,v_N} \varphi_{v_1}(g^{(1)}) \ldots \varphi_{v_N}(g^{(N)})}{\sum_{v_1=0}^{V_1} \cdots \sum_{v_N=0}^{V_N} \tilde{c}_{v_1,\ldots,v_N} \varphi_{v_1}(g^{(1)}) \ldots \varphi_{v_N}(g^{(N)})} \tag{2}$$

The parameter-dependent basis functions $\varphi_{v_i}(g^{(i)})$ are real functions that are associated with a distinct set of poles $\boldsymbol{b}^{(i)} = \{-b_{vi}^{(i)}\}_{v_i=1}^{V_i}$ for each parameter $g^{(i)}$:

$$\varphi_{v_i}(g^{(i)}) = (jg^{(i)} + b_{v_i}^{(i)})^{-1} - (jg^{(i)} - (b_{v_i}^{(i)})^*)^{-1} \tag{3}$$

$$\varphi_{v_{i+1}}(g^{(i)}) = j(jg^{(i)} + b_{v_i}^{(i)})^{-1} + j(jg^{(i)} - (b_{v_i}^{(i)})^*)^{-1} \tag{4}$$

These poles of the basis functions are chosen as complex pairs with small real parts of opposite sign and imaginary parts linearly spaced over the parameter range of interest to ensure a good numerical conditioning. The additional basis function $\varphi_0(g^{(i)}) = 1$ defines the asymptotic behaviour of the model.

If $-a_p$ and $-a_{p+1}$ in (1) constitute a complex conjugate pair of poles, the real and imaginary part of the corresponding residues in (2) are modelled separately. It is remarked that only the poles $\{-a_p\}_{p=1}^{P}$ in (1) must have negative real parts to ensure overall stability of the parametric macromodel. The flipping of unstable poles into the left half plane is used to enforce stability [6]. Note that, regarding the modelling of the parameterised residues, any kind of basis functions can be utilised without affecting the rational nature of the frequency representation.

Three-dimensional example: tapered transmission line: The reflection coefficient S_{11} of a lossless exponential tapered transmission line is revisited from [2] and modelled by the hybrid technique. The line is terminated with a matched load, as shown in Fig. 1, where $Z_0 = 50\ \Omega$ and $Z_L = 100\ \Omega$ represent the reference impedance and the load impedance, respectively [7]. A trivariate macromodel is computed as a function of the varying relative dielectric constant $\varepsilon_r = g^{(1)} \in [3-5]$ and varying line length $L = g^{(2)} \in [1-10]$ cm over the frequency range $f \in [1\text{ kHz}-3\text{ GHz}]$. As an example, the frequency response of the trivariate structure is shown for a fixed value of $\varepsilon_r = 4$ in Fig. 2.

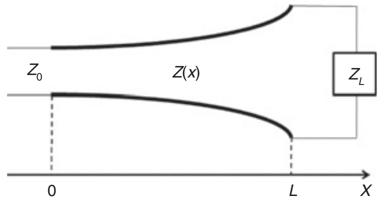

Fig. 1 *Exponential tapered transmission line [7]*

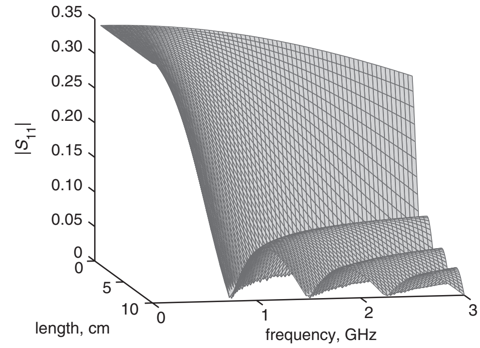

Fig. 2 *Reflection coefficient S_{11} for $\varepsilon_r = 4$*

The initial data is computed over a grid of $12(\varepsilon_r) \times 28(L) \times 32(f)$ samples, and a common set of 14 stable poles is computed, based on all 336 univariate frequency responses. The initial data grid in [2] contains $20(\varepsilon_r) \times 50(L) \times 50(f)$ samples, so the sample density is practically decreased by a factor of 4.65. The parameterised residues $c_p^{(2)}(\varepsilon_r, L)$ corresponding to each pole $-a_p$ are then fitted using $8(\varepsilon_r)$ and $14(L)$ poles, instead of $18(\varepsilon_r)$ and $32(L)$ as in [2].

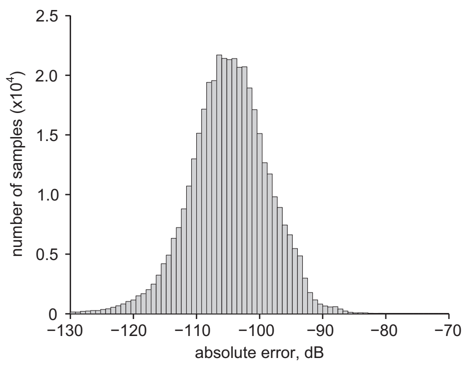

Fig. 3 *Histogram: error distribution over 400 000 validation samples*

To confirm the quality of the built macromodel, it is evaluated and compared over a dense set of $40(\varepsilon_r) \times 100(L) \times 100(f)$ validation samples, and the distribution of the absolute error is shown by a histogram in Fig. 3. It is confirmed that an overall good approximation is obtained, as the maximum error is bounded by −77.6 dB. The required number of model coefficients is reduced from 8128 [2] to 3818 and hence approximately by a factor of 2.13, while preserving the accuracy.

Table 1 recapitulates the advantages of this technique in comparison with [2].

Table 1: Comparison table

	Results from [2]	New approach
Number of coefficients	8128	3818
Number of data samples	50000	10752
Accuracy	−67.56 dB	−77.6 dB
Stability poles	YES	YES

Conclusion: A hybrid method for the macromodelling of parameterised frequency response is presented. It combines the strengths of [2] and [3]: stability of the macromodel is enforced by construction and the required number of model coefficients and data samples is reduced. A numerical example confirms the capability of the method to build compact and accurate parametric macromodels.

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