# Fast Automatic Order Estimation of Rational Macromodels for Signal Integrity Analysis

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#### **Abstract**

A simple rule of thumb is proposed for automatic model order estimation of the rational macromodels for passive components and systems. Based on an analysis of the dynamic behavior of the (measured or simulated) frequency domain scattering parameters, upper and lower bounds for the number of poles are proposed. Several interconnection examples with different complexity are presented to illustrate the usefulness of the proposed approach.

### 1. Introduction

Signal integrity (SI) is a major concern for engineers working on high data rate designs. As the operating frequencies increase, the use of accurate broadband circuit macromodels is paramount to accurately simulate and design complex Giga-bit interconnection systems.

The circuit behavior of passive electrical components and systems can be efficiently described in the frequency domain. Most full-wave electromagnetic (EM) simulation tools (e.g., based on the Method of Moments [1] or Finite Elements [2]) and high-frequency measurement tools (e.g., network analyzers) can accurately simulate or measure the circuit parameters at a set of discrete frequencies over a specified range. These data samples must be condensed into one global macromodel, that is compatible with standard Electronic Design and Automation (EDA) software tools, and that can be used for circuit simulations, design optimization and sensitivity analysis.

The main objective of this paper is to propose a rule of thumb for automatic initial estimation of the order of numerator and denominator of the rational SPICE compatible macromodel, based on the analysis of the dynamic behavior of the scattering parameters.

In the literature, other more advanced and complicated order estimation techniques, pole clustering and pole reduction techniques were proposed [3]-[6] that can be used in combination with our method.

## 2. Iterative rational macromodeling

The Vector Fitting algorithm [7]-[9] is used to identify the residues  $r_n$  and poles  $p_n$  of a rational transfer function R(s),

$$H(s) \approx R(s) = \sum_{n=1}^{N} \frac{r_n}{s - p_n} + H_{\infty}$$
 (1)

such that the difference between  $(s_k, R(s_k))$  and the data samples  $(s_k, H(s_k))$  is minimized in a least squares sense over a predefined frequency range of interest  $[s_0, s_K]$ . The poles are relocated in multiple iterations, and some additional post-

processing steps might be necessary to enforce passivity of the macromodel [10]-[12].

#### 3. Automatic order estimation

In [7], it is mentioned that the number of poles (N) is not of major importance, in that sense that if one takes N too high, the redundant poles will have negligible small residues  $r_n$ , for high S/N ratio data. For typical signal integrity applications, the required number of poles is quite high and it is clear that every extra pole requires an additional non-negligible computational effort, regardless whether its corresponding residue is nearly zero or not. So, it is important to have a good estimate of the required number of poles of the macromodel, especially if this number is quite high.

The proposed adaptive model selection process estimates the order of the macromodel based on the dynamic behavior in the frequency domain of the device under test.

All scattering parameters are displayed as complex values in the smith chart, and the cumulative phase variation along all traces is calculated, by summing all phase variations  $(\Sigma\phi_m)$  between consecutive line segments, between consecutive data samples. The definition of the phase variation is shown on figure 1.

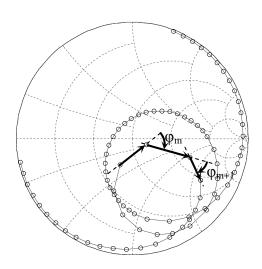


Figure 1: Calculating the cumulative phase variation along the data path, with increasing frequencies.

Phase variations can be ignored if the magnitude of the scattering data is extremely small (e.g., below -80 dB).

Based on physics, it is clear that the consecutive phase variations will always be negative (i.e., vary clockwise) as the frequency increases. If the phase variation is not negative, the data might be undersampled or might be contaminated by

noise. If the data is undersampled (e.g., near a resonance frequency), the corresponding corrected phase variation must be taken into account. Otherwise, if the data is contaminated by noise, some kind of data filtering can be used upfront (e.g., a moving average algorithm).

Once the cumulative phase variation (or "number of turns") of the S-data is determined, an initial estimation of the lower and upper bounds of the required number of poles (*N*) is given by multiplying the maximum (with regard to all S-parameters) "number of turns" by 3 and 4, respectively, for RMS error levels of about -60 dB.

Then, rational macromodels (see eq. 1) of different complexity can be built and evaluated, from simple to complex (i.e., "bottom-up") or the other way around (i.e., "top-down") [3]-[4]. Once all accuracy requirements are met, the adaptive modeling algorithm can stop.

Note that, in the case of a pure delay e<sup>-sT</sup>, the rational Padé approximation [12] of the cumulative phase variation for "1 turn" in the smith chart, using 4 poles, results in an RMS error of 0.0045.

## 4. Validation examples

For evaluation purposes, we estimate the order of the rational macromodels of three datasets with different properties in terms of the amount of delay, reflection and coupling. These reference examples are described in detail in [13].

## 4.1. Single package via

The "single package via" structure has a small delay, small loss, no coupling, and small reflection (50 Ohm system). The 2-port scattering parameters are obtained from full-wave EM simulations (from DC to 20 GHz).

The smith chart of the most dynamic S parameter is shown in figure 2. The maximum cumulative phase variation corresponds to 1.1 turns in the smith chart. The RMS error of the 4-pole rational macromodel is 0.0034. Note that the RMS accuracy is defined as the worst case RMS over all S-parameters.

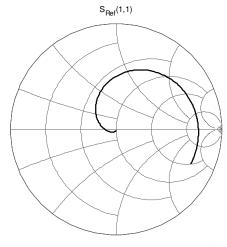


Figure 2: Single package via: smith chart of S(1,1) (50 Ohm ref).

## 4.2. Coupled microstrips

The "coupled microstrips" structure has a large delay, significant coupling, but low loss and no reflection (50 Ohm system). The 4-port system is measured using a vector network analyzer (from 50 MHz to 20 GHz).

The smith chart of the S(2,1) parameter is shown in figure 3. The maximum cumulative phase variation corresponds to 39.4 turns in the smith chart. The RMS error of the 120-pole rational macromodel is 0.0016.

In figure 4, the magnitude of the first column of the S-matrix is shown (measured data and macromodel). From figure 5, it is clear that the difference between the macromodel and the measured input data is comparable with the noise level of the measurements.

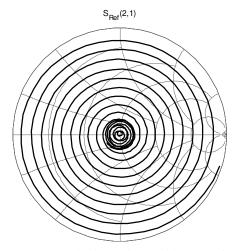


Figure 3: Coupled microstrips: smith chart of S(2,1) (50 Ohm ref).

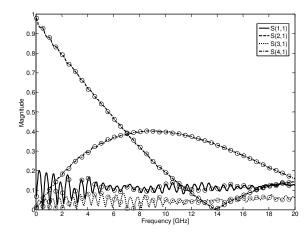


Figure 4: Coupled microstrips: magnitude of the measured data (drawn as lines) and macromodel (indicated with markers).

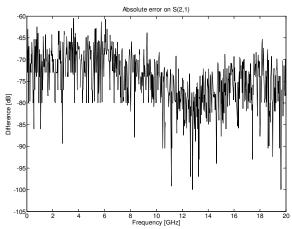


Figure 5: Coupled microstrips: difference between macromodel and measured data of the transmission parameter S(2.1).

### 4.3. RDRAM channel

The "RDRAM channel" data has a large delay, significant coupling and important reflection (28 Ohm system). The 4-port memory channel is measured using a vector network analyzer (from 50 MHz to 2.5 GHz).

The smith chart of the S(3,3) parameter is shown in figure 6. The maximum cumulative phase variation corresponds to 15.5 turns in the smith chart. The RMS error of the 47-pole rational macromodel is 0.0013.

In figure 7, the magnitude of the third column of the S-matrix is shown (measured data and the macromodel).

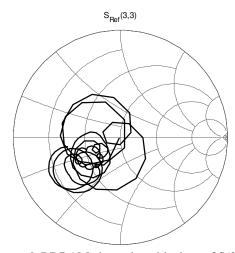


Figure 6: RDRAM channel: smith chart of S(3,3) (50 Ohm ref).

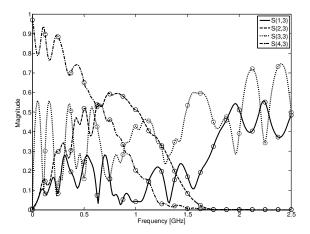


Figure 7: RDRAM channel: magnitude of the measured data (drawn as lines) and macromodel (indicated with markers).

#### 5. Conclusions

A rule of thumb, that can be easily and rapidly evaluated, is proposed for automatic model order estimation of the rational macromodels. Based on an analysis of the dynamic behavior of the (measured or simulated) frequency domain scattering parameters, upper and lower bounds for the number of poles are proposed. As demonstrated, the method can be used for systems with various complexity.

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