Adaptive Knot Placement for Rational Spline Interpolation of Sparse EM-Based Data

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Abstract

Frequency domain interpolation methods suffer poor numerical conditioning when the frequency range or the model order becomes large. Therefore, an earlier developed modeling technique, called AFS [1], is extended by using smart piecewise rational interpolation with adaptive knot placement. The AFS algorithm is used to model the spectral response of general passive planar electrical structures over a frequency range of interest, based on a limited number of data samples. The new algorithm adaptively chooses an appropriate sample distribution, while minimizing the complexity of the model and the number of splines, without any prior knowledge of the dynamics of the system.

1. INTRODUCTION

In a previous paper [1], an Adaptive Frequency Sampling (AFS) algorithm was introduced, to model the spectral response of general passive planar electrical structures over a frequency range of interest. The numerator and denominator of the rational transfer function were decomposed in a separate basis of Forsythe polynomials that are orthogonal on the inner product defined by the normal equations of the estimator [2]. This method -combined with an appropriate frequency scaling- was used to create rational fits, since it makes the set of normal equations best conditioned [3]-[4]. However, components which are sampled over a very broad frequency range, or require a high model complexity, still can't be modeled with sufficient accuracy, because of numerical problems.

Therefore, the algorithm is extended with piecewise rational interpolation. Instead of splitting the frequency range of interest in an ad hoc number of subranges, which are each modeled separately, a technique is presented to minimize the number of splines in an adaptive way, based on the numerical conditioning of the system.

2. ENHANCED AFS ALGORITHM

2.1 Rational model

The EM-based data is approximated by a rational function with a Forsythe orthogonal based numerator and denominator:

$$H[\hat{s}_i] = \frac{\sum_{n=0}^{N} N_n p_n(\hat{s}_i)}{D_0 + \sum_{d=1}^{D} D_d p_d(\hat{s}_i)} \quad \text{with } \hat{s}_i = j2\pi\hat{f}_i \qquad (1)$$

where $H[s_i]$ represents the S-parameter data samples simulated at discrete complex frequencies s_i, $\forall i = 1,\dots,k$. Scaled frequencies are denoted with a carat, and $p_j(\hat{s}_i)$ represents the monic orthogonal Forsythe polynomial of order j. The three-term recurrence relation of the orthogonal Forsythe polynomials is given by:

$$\begin{aligned}
&p_0(\hat{s}_i) = 1 \\
&p_1(\hat{s}_i) = \hat{s}_i \cdot p_0(\hat{s}_i) - \alpha_1 \cdot p_0(\hat{s}_i) \\
&p_2(\hat{s}_i) = \hat{s}_i \cdot p_1(\hat{s}_i) - \alpha_2 \cdot p_1(\hat{s}_i) - \beta_1 \cdot p_0(\hat{s}_i) \\
&\dots \\
&p_{j+1}(\hat{s}_i) = \hat{s}_i \cdot p_j(\hat{s}_i) - \alpha_{j+1} \cdot p_j(\hat{s}_i) - \beta_j \cdot p_{j-1}(\hat{s}_i)
\end{aligned} \qquad (2)$$

where the α and β coefficients are calculated directly from the scaled frequencies, to obtain best conditioning of the normal equations.

The coefficients can be calculated by solving a linear least squares problem:

$$\mathbf{X} \cdot \begin{pmatrix} N_0 \\ \dots \\ N_N \\ D_1 \\ \dots \\ D_D \end{pmatrix} = \begin{pmatrix} \Re(-p_0(\hat{s}_0)H[\hat{s}_0]) \\ \Im(-p_0(\hat{s}_0)H[\hat{s}_0]) \\ \dots \\ \dots \\ \Re(-p_0(\hat{s}_0)H[\hat{s}_{k-1}]) \\ \Im(-p_0(\hat{s}_0)H[\hat{s}_{k-1}]) \end{pmatrix} \qquad (3)$$

where X equals:

$$\begin{pmatrix} \Re(-p_0(\hat{s}_0)) & \ldots & \Re(-p_N(\hat{s}_0)) & \Re(p_1(\hat{s}_0)H[\hat{s}_0]) & \ldots & \Re(p_D(\hat{s}_0)H[\hat{s}_0]) \\ \Im(-p_0(\hat{s}_0)) & \ldots & \Im(-p_N(\hat{s}_0)) & \Im(p_1(\hat{s}_0)H[\hat{s}_0]) & \ldots & \Im(p_D(\hat{s}_0)H[\hat{s}_0]) \\ \ldots & & \ldots & \ldots & & \ldots \\ \Re(-p_0(\hat{s}_{k-1})) & \ldots & \Re(-p_N(\hat{s}_{k-1})) & \Re(p_1(\hat{s}_{k-1})H[\hat{s}_{k-1}]) & \ldots & \Re(p_D(\hat{s}_{k-1})H[\hat{s}_{k-1}]) \\ \Im(-p_0(\hat{s}_{k-1})) & \ldots & \Im(-p_N(\hat{s}_{k-1})) & \Im(p_1(\hat{s}_{k-1})H[\hat{s}_{k-1}]) & \ldots & \Im(p_D(\hat{s}_{k-1})H[\hat{s}_{k-1}]) \end{pmatrix}$$

2.2 Enhanced AFS Algorithm

The flow chart of the new enhanced adaptive algorithm is shown in figure 1. It consists of an adaptive modeling loop, an adaptive spline loop, and an adaptive sample selection loop.

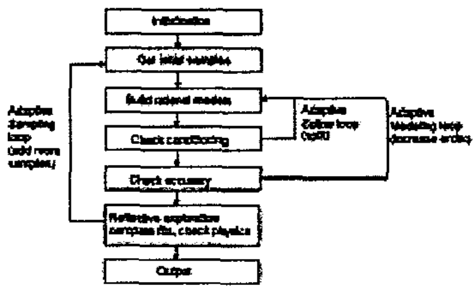

Fig. 1 : Flowchart of the enhanced AFS algorithm

The algorithm starts with four samples equidistantly spread over a certain frequency range of interest. Depending on the number of available data samples, multiple rational models are built with different order of numerator and denominator, exploiting all degrees of freedom.

The rational fitting models are evaluated in the data points, and compared against one another. If the error between the rational model evaluated in the selected data points and the simulated s-parameters, exceeds a certain threshold (*e.g.* –80 dB), the model is rejected, and the model's complexity is increased.

The difference between the two models is called the estimated fitting error, and new samples should be chosen in such way, that the maximum estimated fitting error is minimized. A reliable way to estimate the fitting error, and select new samples is given in [1].

The process of selecting data samples and building models adaptively, is called *reflective exploration,* which is useful when the process that provides the data is very costly, which is the case for full-wave EM simulators.

Usually, up to 20 samples, the algorithm works fine, however if a higher model complexity is needed, the linear least squares system becomes ill-conditioned. Splitting the frequency range a priori raises the question how many splines will be needed to reach the desired accuracy without numerical problems. Since there is no information about the system's dynamics available, and the desired sample distribution is usually not uniform, this method can't lead to optimal results.

Splitting the frequency range at run-time, when the number of selected samples exceeds a certain threshold, is another possible approach, although it is also inefficient and can lead to undesired behavior. When the static threshold is set too high and numerical problems aren't resolved until afterwards, the last samples that were selected may not be optimal. When it is set too low, the number of splines may grow larger than necessary, and require extra data samples that could be avoided.

A better approach is to check the condition number of the matrix X (3), each time a new sample is added, and the model's complexity is increased. The splitting occurs only when the condition number exceeds a threshold. This way, the algorithm can guarantee the number of significant digits, and the number of splits is minimized.

If the system is ill-conditioned, the frequency range is split in 2 subranges which share one common data sample. Instead of splitting the frequency range in two subranges that are equally large, information from the available samples should be used.

The knot is placed at the mean of the frequencies of the selected samples, and in such way that each spline covers at least one third of the original frequency range. Since the mean of the selected samples is often located around resonance frequencies or where important coupling effects occur, and where a lot of samples are chosen, the spline knot is usually placed where the s-parameters are changing rapidly.

The right spline is considered first, and the process of adaptively adding more samples, increasing the model complexity, and splitting in subranges is repeated iteratively until one part of the frequency range is modeled accurately.

Then, secondly, all subranges of the left splines, and consequently all the corresponding samples that were already selected, are merged into one large interval. And the adaptive modeling process is iteratively repeated, until all subranges are modeled.

2.3 Example - Multipole Filter

A multipole component with high complexity is modeled with the enhanced AFS modeling technique. The smart knot placement of the splines is demonstrated in Fig. 2.a-2.k.

The figures illustrate the different steps of iterative sampling, modeling and the knot placing process of the reflection coefficients of a multipole filter. The algorithm uses only 87 data samples and 5 rational splines, to reach the desired accuracy.

The original data is represented by a solid line, and the interpolation data is dotted. Crosses refer to samples. Circles refer to spline knots.

3. CONCLUSION

An adaptive modeling method is extended with spline interpolation and a smart knot placement technique, based on the condition number of the fit. Multiple rational models were used to interpolate S-parameter data, obtained through electro-magnetic simulations.

The adaptive algorithm doesn't require any prior knowledge of the system's dynamics to select an appropriate sample distribution, spline knot distribution and an appropriate model complexity.

The algorithm avoids oversampling and undersampling, as well as overmodeling and undermodeling, and guarantees numerical stable results.

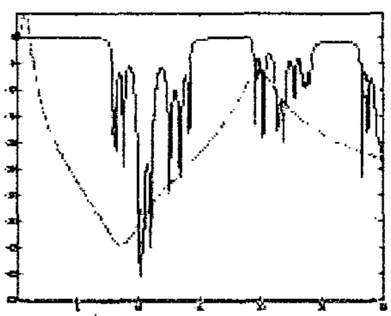

Fig. 2.a mag(S_{11}) based on 4 samples

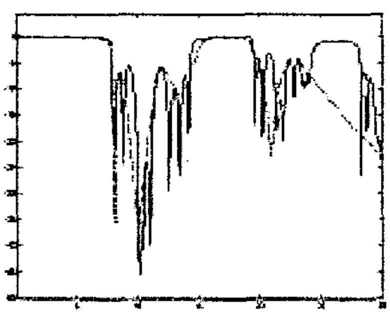

Fig. 2.b mag(S_{11}) based on 20 samples

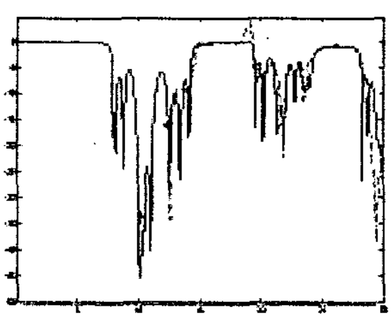

Fig. 2.c mag(S_{11}) based on 22 samples

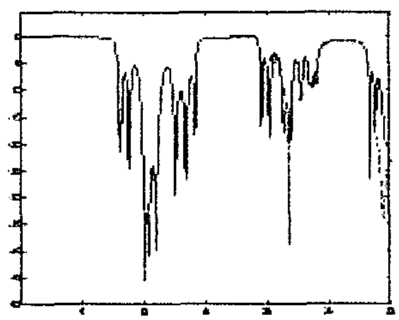

Fig. 2.d mag(S_{11}) based on 18 samples

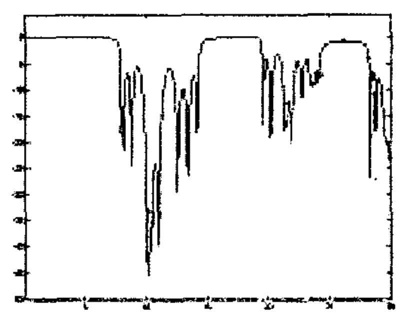

Fig. 2.e mag(S_{11}) based on 18 samples

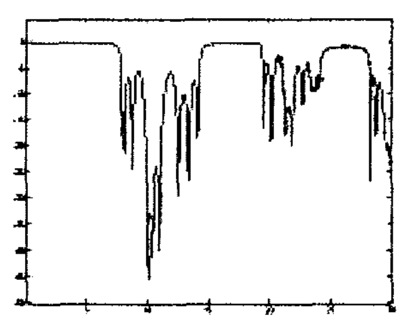

Fig. 2.f mag(S_{11}) based on 37 samples

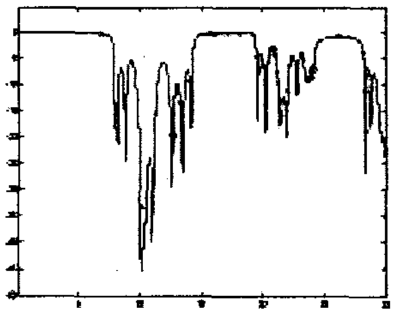

Fig. 2.g mag(S_{11}) based on 56 samples

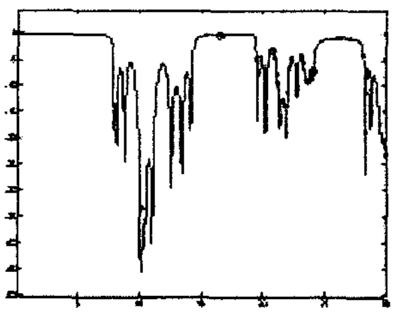

Fig. 2.h mag(S_{11}) based on 49 samples

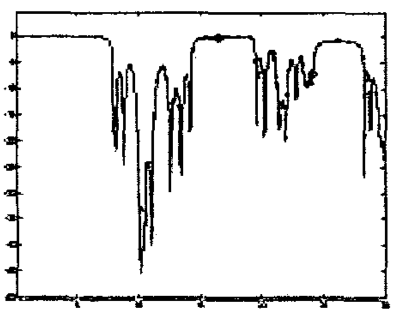

Fig. 2.i mag(S_{11}) based on 67 samples

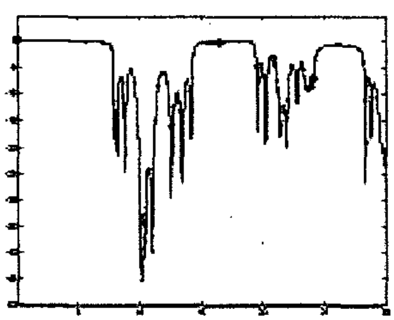

Fig. 2.j mag(S_{11}) based on 87 samples

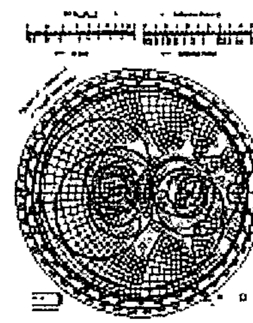

Fig. 2.k : Smith chart

4. REFERENCES

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