

Rational Fitting of S-Parameter Frequency Samples With Maximum Absolute Error Control

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Abstract—Rational fitting techniques are often used for the macromodeling of linear systems from tabulated S-parameter frequency samples. This letter proposes a modified weighting scheme for the Vector Fitting algorithm that iteratively minimizes the maximum absolute error over the frequency range of interest, rather than the least-squares error. By considering the appropriate error measure in the fitting process, it is possible to obtain more accurate broadband macromodels without increasing the number of poles. The effectiveness of the approach is illustrated by several numerical examples.

Index Terms—Error control, least squares, macromodeling, rational approximation, S-parameters, vector fitting.

I. INTRODUCTION

FREQUENCY domain macromodeling techniques are becoming increasingly important for the design, study and optimization of microwave structures. The well-known Vector Fitting algorithm [1] has been widely recognized as a robust technique to compute such macromodels in an efficient way, and it has been adopted by several engineering communities.

Vector Fitting computes a rational macromodel by iteratively minimizing the least-squares distance between a discrete set of frequency samples and the calculated rational macromodel. However, when dealing with S-parameter frequency samples, it is often preferable to minimize the maximum absolute error over the frequency range of interest. Therefore, it is clear that a standard application of Vector Fitting minimizes a different error measure than the one which is desired.

This letter modifies the weighting scheme of the Vector Fitting algorithm in such a way that the maximum absolute error is minimized. By applying a suitable error-based weighting scheme to the frequency samples in each iteration step, it is possible to compute macromodels that are tailored towards the error measure of interest. Several numerical examples illustrate the usefulness and benefits of this approach.

II. VECTOR FITTING ALGORITHM

Given a set of S-parameter samples $\{s_k, H(s_k)\}_{k=1}^K$, the Vector Fitting algorithm [1] computes a rational macromodel that matches the frequency response by solving several least squares problems in successive iteration steps ($t = 0, \dots, T$)

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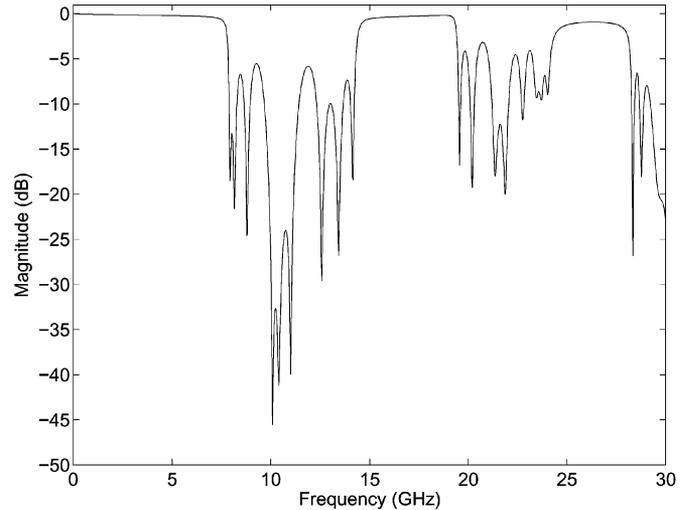


Fig. 1. Multipole filter: Magnitude of frequency response.

$$\arg \min |(\sigma H)^t(s) - \sigma^t(s)H(s)|^2. \quad (1)$$

Both $(\sigma H)^t(s)$ and $\sigma^t(s)$ in (1) are expanded as a linear combination of partial fractions with common poles a_p^t

$$(\sigma H)^t(s) = \sum_{p=1}^P \frac{c_p^t}{s - a_p^t} + c_0^t \quad (2)$$

$$\sigma^t(s) = \sum_{p=1}^P \frac{\tilde{c}_p^t}{s - a_p^t} + \tilde{c}_0^t. \quad (3)$$

Note that in the first iteration step ($t = 0$), the initial starting poles a_p^0 are selected according to a heuristical scheme [1]. It is trivial to transform (1) into an overdetermined set of least-squares equations $A^t x^t = b^t$, where the solution vector x^t contains the unknown coefficients $\{c_p^t, \tilde{c}_p^t\}_{p=0}^P$. Based on the coefficients $\{\tilde{c}_p^t\}$, it is possible to obtain a new set of relocated poles by solving the zeros of $\sigma^t(s)$ as an eigenvalue problem. These relocated poles replace the initial set of poles, and the process of solving (1) is repeated iteratively until the poles are converged to some quasi-optimal position. The trivial null solution of (1) is avoided by setting one coefficient $\tilde{c}_0^t = 1$ or by adding an additional relaxation constraint to (1), as in [2].

In the final iteration step ($t = T$), the transfer function $(\sigma H)^T(s)$ in partial fraction form is obtained by solving (1) with $\sigma^T(s) = 1$. All the details about this Vector Fitting procedure are extensively reported in literature, see [1]–[6].

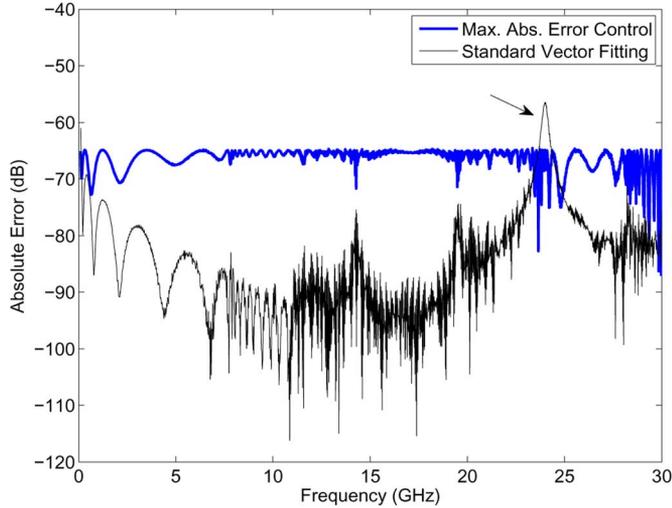


Fig. 2. Multipole filter: Absolute error of macromodel.

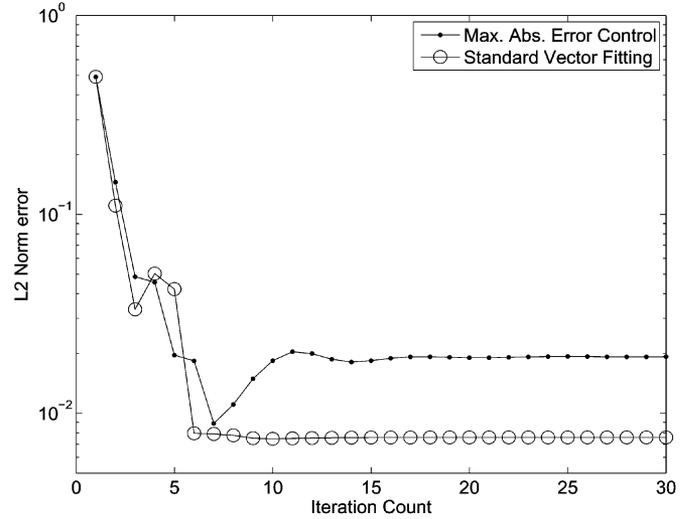


Fig. 4. Multipole filter: Comparison of L2 norm error.

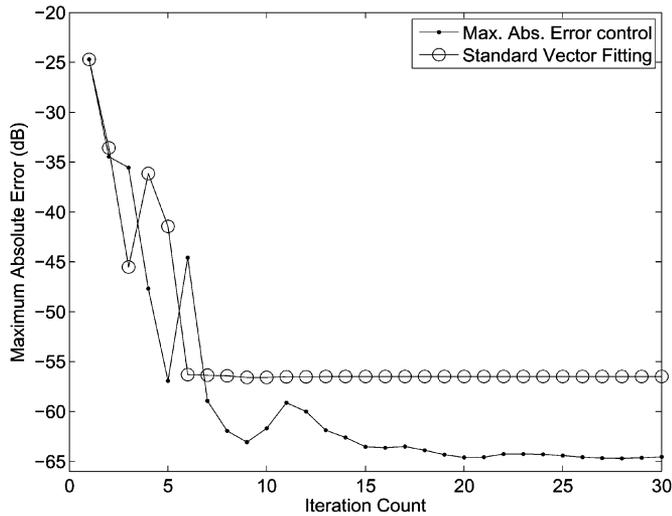


Fig. 3. Multipole filter: Comparison of maximum absolute error.

III. MAXIMUM ABSOLUTE ERROR CONTROL

In order to obtain maximum absolute error control, the least-squares problem (1) is modified by introducing a user-defined frequency-dependent weighting function $w^t(s)$ as follows:

$$\arg \min |w^t(s)|^2 |(\sigma H)^t(s) - \sigma^t(s)H(s)|^2. \quad (4)$$

It is well known that frequency-dependent weighting schemes are a powerful way of controlling the accuracy of the macromodel. In the Vector Fitting algorithm, the value of $w^t(s)$ is often set to 1 by default. However, in some cases, inverse magnitude weighting has also been considered to fit frequency samples with an accuracy that is relative to their magnitude. In such cases, the value of these weights is often determined by the characteristics (such as e.g., the magnitude) of each frequency sample individually, leading to *local* error control.

In order to minimize the maximum absolute error over all frequency samples, a *global* approach is needed that takes the overall accuracy of the model into account. This can be obtained

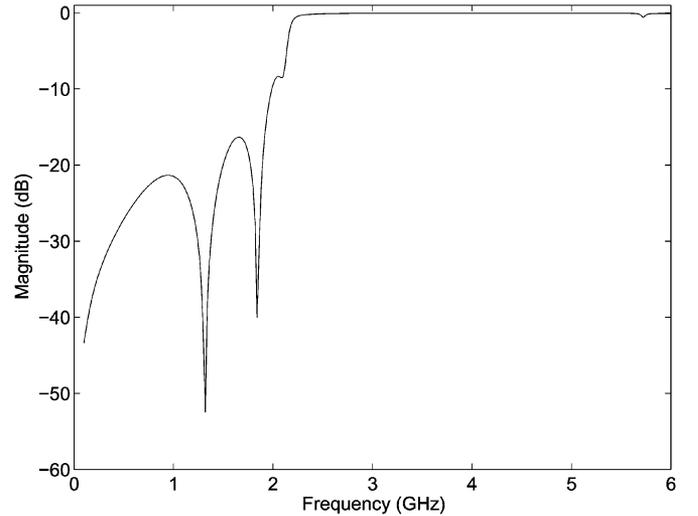


Fig. 5. Bandpass filter: Magnitude of frequency response.

by updating the weights in (4), based on the absolute error of the macromodel, in each iteration step [7]

$$w^t(s) = w^{t-1}(s) \sqrt{\left| \frac{(\sigma H)^{t-1}(s)}{\sigma^{t-1}(s)} - H(s) \right|}. \quad (5)$$

It is clear from (5) that frequency samples with a large absolute error are assigned a higher weight than other frequency samples. It is also noted that the weighting function $w^t(s)$ can be used as a replacement for the Sanathanan-Koerner weighting [7]–[9], which here occurs implicitly by pole relocation [4].

IV. EXAMPLE: MULTIPOLE MICROSTRIP FILTER

The reflection coefficient S_{11} of a two-port dispersive multipole microstrip filter is simulated over the frequency range of interest [0.1 GHz–30 GHz]. The fast relaxed Vector Fitting algorithm [1]–[3] is then applied to approximate the S-parameter data by a 56-pole strictly proper transfer function in 30 iteration steps [10]. The desired accuracy is a maximum absolute error

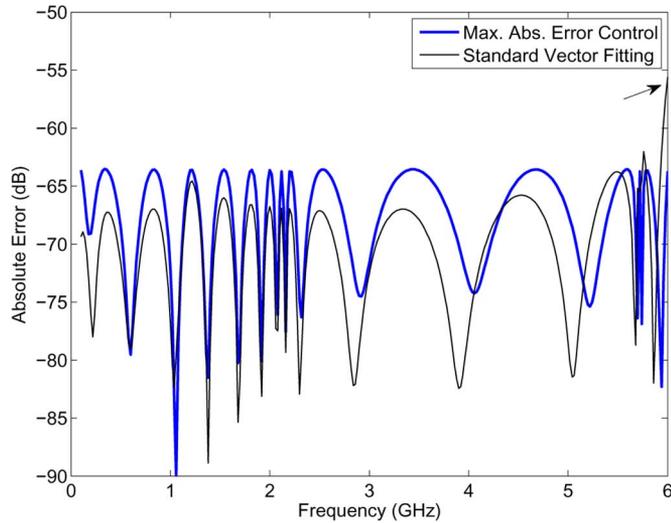


Fig. 6. Bandpass filter: Absolute error of macromodel.

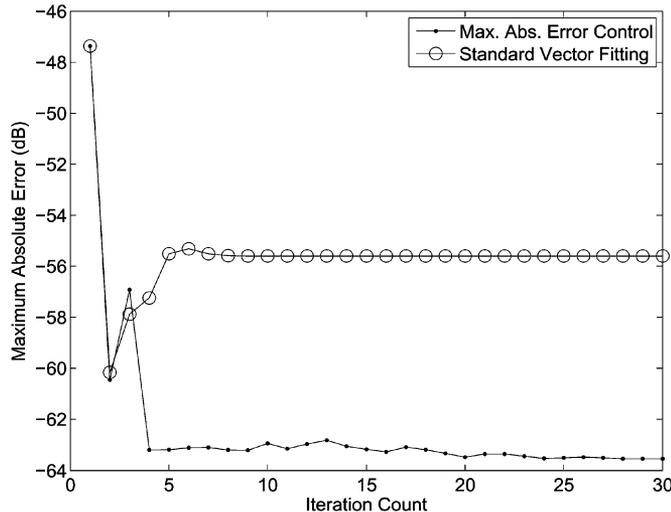


Fig. 7. Bandpass filter: Comparison of maximum absolute error.

of -60 dB, which corresponds to the accuracy level of the simulator (i.e., approximately 3 significant digits).

The magnitude of the frequency response is shown in Fig. 1 and the corresponding absolute error is shown in Fig. 2. It is found that the maximum absolute error using the standard Vector Fitting implementation equals -56.48 dB (see arrow). However, if the error-based weighting scheme in Section III is applied, then the maximum absolute error is reduced to -64.53 dB, which is significantly more accurate. It is seen from Fig. 2 that this gain in accuracy is obtained by spreading the overall absolute error more uniformly over the frequency range of interest, at the expense of a larger least-squares fitting error. This is confirmed by Figs. 3 and 4, showing the evolution of both error measures as a function of the iteration steps.

It is possible to obtain a similar accuracy using the standard Vector Fitting without error control, although this requires an additional amount of poles. The use of these additional poles is often undesired, since it leads to larger equivalent circuits and increases the probability of outband passivity violations.

V. EXAMPLE: BLUETOOTH LTCC BANDPASS FILTER

As a second example, the reflection coefficient S_{11} of a two-port Bluetooth LTCC bandpass filter is simulated over the frequency range $[0.1 \text{ GHz} - 6 \text{ GHz}]$. The magnitude of the data is shown in Fig. 5, and the S-parameters are approximated by a 16-pole strictly proper transfer function in 30 iteration steps.

It is found that the maximum absolute error using the standard Vector Fitting implementation is equal to -55.59 dB, due to a larger deviation at the higher frequencies (see arrow in Fig. 6). If the error-based weighting scheme is applied, then the maximum absolute error can be reduced to -63.53 dB. As can be seen from Fig. 6, the absolute error becomes spread more uniformly over the frequency range of interest.

The evolution of the maximum absolute error as a function of the iteration steps is shown in Fig. 7. These results confirm the practical usefulness of the proposed weighting scheme.

VI. DISCUSSION

It is known that iterative weighting schemes do not guarantee convergence or a monotonically decreasing error in each iteration step. Therefore, the evolution of the error should be monitored during the iterations, and the algorithm is terminated if the solution does not lead to further improvements.

VII. CONCLUSION

A new frequency-dependent weighting scheme is applied to the Vector Fitting algorithm, which allows maximum absolute error control during the fitting process. This is particularly useful for the macromodeling of S-parameter frequency samples. Two numerical examples confirm that better results can be obtained if the weighting is tailored towards the error measure of interest.

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