On the Application of Dimensional Analysis to Parametric Macromodeling

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***Abstract*—Fast parametric macromodeling techniques are important for the design and analysis of microwave structures. Standard approaches in the literature are often limited to problems with a moderate amount of parameters, since the complexity of the problem grows exponentially with the number of dimensions. This letter investigates the use of dimensional analysis as a means to reduce the dimensionality of the approximation problem.**

***Index Terms*—Dimensional analysis, least squares, parametric macromodeling, surface approximation, vector fitting.**

I. Introduction

PARAMETRIC macromodels are very important for fast design, study and optimization of broadband microwave structures. Such macromodels approximate the frequency domain behavior of a system as a function of the frequency and one (or several) design parameters. It is however known that the complexity of the macromodeling process grows exponentially with the number of dimensions, and thus the applicability of parametric macromodeling techniques is often limited to systems with a moderate amount of parameters. This property is often referred to as "the curse of dimensionality" [1]–[6].

This letter investigates the use of dimensional analysis as a method for reducing the number of macromodel parameters [7]–[9] . Dimensional analysis relies on the fundamental principle that any relation between the model parameters must be dimensionally consistent. This information can be exploited to compute parametric macromodels with a lower dimensional complexity, while preserving all the information of the original design. In some cases, this can lead to significant savings in terms of computation time and memory resources. The usefulness of the approach is illustrated by two examples.

II. Dimensional Analysis

In dimensional analysis, Buckingham's Π-theorem states that every physically meaningful equation involving N dimensional parameters can be equivalently rewritten as an equation of $N_r = N - D$ dimensionless π -parameters, where D is the number of fundamental dimensions used. These dimensionless π-parameters are formed by multiplication of dimensional parameters raised to powers that make the product dimensionless. The concept of dimensional analysis is well described in the literature (see e.g., [10]) and a short outline of the procedure is given for convenience of the reader:

1) All influential design parameters in the problem (including frequency) are listed and expressed in terms of their basic dimensions. Note that the frequency response (i.e., output) of the system is also considered as a parameter.
2) Then, one counts the number of parameters N and the number of independent dimensional constraints D. The independence of the parameters can be determined by considering the rank of the dimensional matrix.
3) One forms $N_r = N - D$ dimensionless π-parameters by multiplication of the dimensional parameters raised to the proper power. So, the formed parameters cannot be written as a linear combination of each other. Note that the π-parameters are not always uniquely defined.
4) The general relationship among the π-parameters can then be expressed as $\pi_1 = R(\pi_2, \pi_3, \ldots, \pi_{N_r})$.

III. Parametric Macromodeling

Once the dimensionality of the problem has been reduced by following the steps in Section II, one can use, for example, the Multivariate Orthonormal Vector Fitting (MOVF) technique [5] to compute a parameterized rational macromodel R that approximates the system response of the structure as a function of the dimensionless π-parameters

$$\pi_1 = R(\pi_2, \pi_3, \ldots, \pi_{N_r}). \quad (1)$$

It is clear that this macromodeling technique can be applied directly as in [5] without further modifications. However, since only $N_r < N$ parameters are involved in the macromodeling process, the fitting can be computationally more efficient.

IV. Example: Tapered Transmission Line

As an example, dimensional analysis is applied to model the reflection coefficient $\Gamma_{\rm in}$ of a lossless exponential tapered transmission line terminated in matched loads. The response is computed as a function of 3 dimensional parameters: angular frequency ω, dielectric constant ε and line length l. This corresponds to the example reported in Section X of [5].

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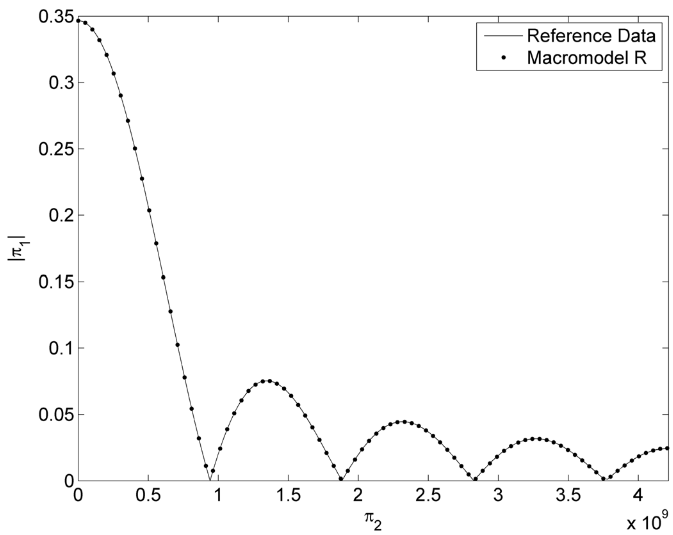

Fig. 1. Tapered TML : magnitude of π_1 as a function of π_2 (dimensionless).

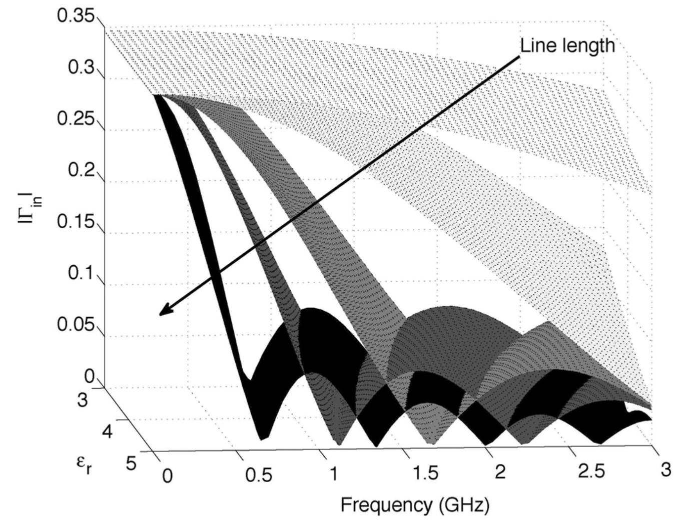

Fig. 2. Tapered TML : magnitude of reflection coefficient $\Gamma_{\rm in}$ as a function of ω, l, ε and fixed μ (dimensional).

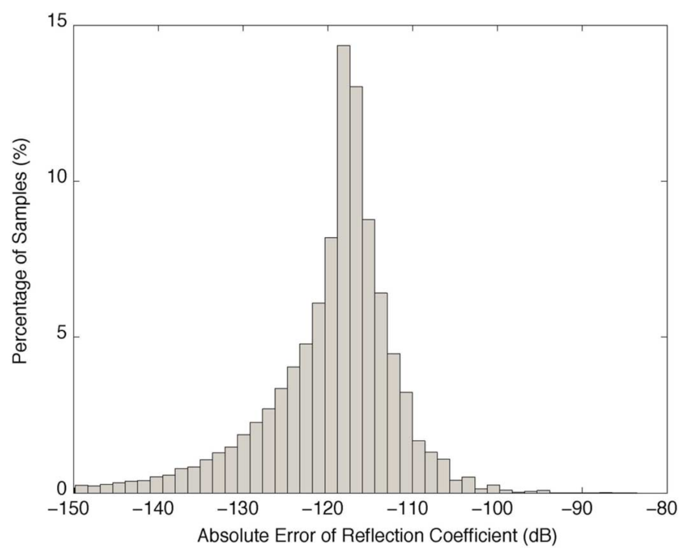

Fig. 3. Tapered TML : histogram of error distribution over dense set of 13125 validation samples.

To make the parameters dimensionally homogeneous, one also needs to take the permeability μ into account, as a constant. The $N = 5$ dimensional parameters and their dimensions are listed in Table I, and $D = 3$ independent dimensions are identified from the dimensional matrix. Thus, one can construct $N_r = 2$ dimensionless π-parameters

$$\pi_1 = \Gamma_{\rm in} \quad \pi_2 = \omega l \sqrt{\varepsilon\mu}. \tag{2}$$

In the dimensionless case, fitting the reflection coefficient $\Gamma_{\rm in}$ leads to a 1-dimensional approximation problem that can be solved using univariate Vector Fitting [11] with pole flipping

$$\Gamma_{\rm in} = \pi_1 \approx R(\pi_2) = \sum_{p=1}^{P} \frac{c_p}{j\pi_2 - a_p} + d. \tag{3}$$

The number of poles P is chosen equal to 12. To obtain the dimensional model, (3) can be reformulated as follows:

$$\Gamma_{\rm in} = \pi_1 \approx R(\omega, l, \varepsilon, \mu) = \sum_{p=1}^{P} \frac{(c_p/l\sqrt{\varepsilon\mu})}{j\omega - (a_p/l\sqrt{\varepsilon\mu})} + d. \tag{4}$$

Fig. 1 shows the magnitude of $\Gamma_{\rm in}$ and the univariate macromodel (3) in the dimensionless π-parameter formulation, and it is seen that an excellent agreement is obtained. The response of the dimensional parametric macromodel (4) with variables ω, l, ε and fixed μ is then evaluated on a grid and compared to a dense set of 13125 validation samples. A visualization of the response is shown in Fig. 2, and the distribution of the absolute error is shown by a histogram in Fig. 3. It turns out that the maximum absolute error is -83 dB, which is comparable to the result in [5]. Note however that model (4) is more compact, and requires much less computation time and memory resources to compute when compared to [5].

TABLE I
PARAMETERS OF TRANSMISSION LINE EXAMPLE

Parameter	Dimension	Range	Description
ω	$1/s$	$(0.001-3)2\pi$ GHz	Angular Freq.
l	m	$(1-10)$ cm	Line length
ε	$s/m\Omega$	$(3-5)\varepsilon_0$	Permittivity
μ	$s\Omega/m$	$(1)\mu_0$	Permeability
Γ_{in}	-	-	Reflection

V. EXAMPLE: OPEN-ENDED STRIPLINE

As a second example, dimensional analysis is applied to model an open-ended stripline. All data samples are simulated with ADS Momentum [12]. The geometry of the stripline is shown in Fig. 4, and an overview of all the parameters with their dimensions and parameter ranges is shown in Table II. The response is computed as a function of 5 dimensional parameters: all the geometric parameters (i.e., length l, width w and height h), the angular frequency ω and the dielectric constant

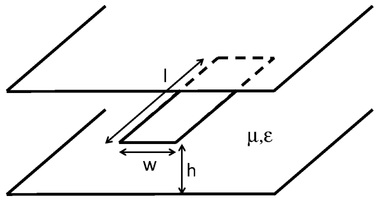

Fig. 4. Geometry of open-ended stripline.

TABLE II
PARAMETERS OF OPEN-ENDED STRIPLINE EXAMPLE

Parameter	Dimension	Range	Description
ω	$1/s$	$(0.1-3)2\pi$ GHz	Angular freq.
l	m	$(20-25)$ mm	Line length
w	m	$(1.1-1.3)$ mm	Line width
h	m	$(0.4-0.6)$ mm	Substrate height
ε	$s/m\Omega$	$(2-3)\varepsilon_0$	Permittivity
μ	$s\Omega/m$	$(1)\mu_0$	Permeability
Z_{in}	Ω	-	Port Impedance
S_{11}	-	-	Reflection

ε. Similarly as in the previous example, the set is extended with the constant permeability μ. Also the reference impedance Z_{in} of the input port must be taken into account.

Following the different steps of the dimensional analysis as described in Section II, $N_r = 5$ dimensionless π-parameters are formed (since $N = 8$ and $D = 3$)

$$\pi_1 = S_{11}, \quad \pi_2 = w/h, \ \pi_3 = l/h, \tag{5}$$

$$\pi_4 = \omega l\sqrt{\varepsilon\mu}, \pi_5 = Z_{\text{in}}\sqrt{\varepsilon/\mu}. \tag{6}$$

In the following, π_5 is always kept constant, and it is straightforward to calculate the results for other values of Z_{in} using standard analytic formulae. Since π_1 is the reflection coefficient S_{11} (output) and π_5 is constant, it follows that the fitting leads to a 3-dimensional (instead of a 5-dimensional) approximation problem that can be solved using MOVF [5]. The number of poles corresponding to each dimensionless parameter is set to 2 (π_2), 2 (π_3) and 6 (π_4), respectively, and an accurate macromodel is obtained in 2 iteration steps.

To assess the quality of the overall model, the response is compared to a set of 7936 validation samples and the distribution of the error is shown by a histogram in Fig. 5. It is seen that the maximum absolute error is −67 dB, which corresponds to the noise level of the simulator [12].

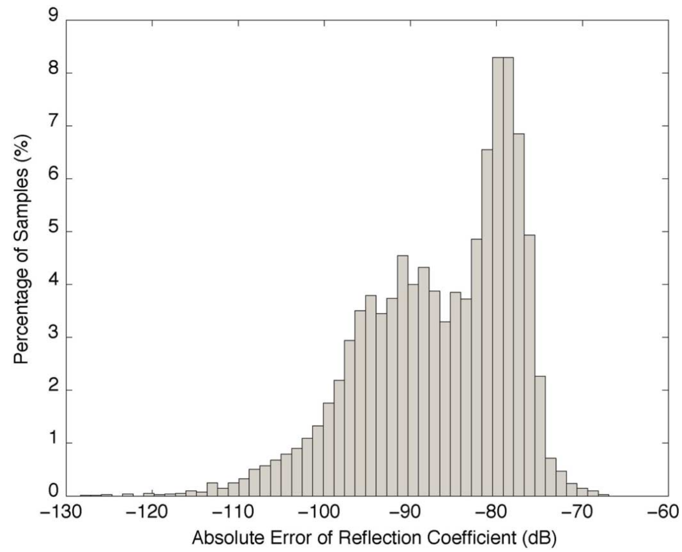

Fig. 5. Open-ended stripline : histogram of error distribution over dense set of 7936 validation samples.

VI. DISCUSSION

It is noted that not every set of parameters can be subjected to dimensional analysis, since the problem needs to preserve dimensional homogeneity. Nevertheless, this is most often the case when dealing with electromagnetic problems. Also, Buckingham's Π-theorem states that a complete set of influential parameters must be considered, even if they are kept constant in the original design. Such a requirement can limit the reduction factor, but in the worst case, no more parameters than in the original design are obtained. This property makes dimensional analysis useful for variability investigations, when the modeling involves a large number of parameters.

VII. CONCLUSION

This letter applies dimensional analysis to the parametric macromodeling of microwave structures. It is known that the dimensionality of such macromodels grows exponentially with the number of parameters. Despite some limitations, it is shown by two numerical examples that dimensional analysis can easily be applied to reduce the dimensionality of the approximation problem. In some cases, this can lead to a more efficient and effective macromodeling procedure.

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