Passivity-Preserving Interpolation-Based Parameterized Macromodeling of Scattered S-Data

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***Abstract*—A new technique to build parametric macromodels based on scattered S-data samples in the design space is presented. Stability and passivity are guaranteed by construction over the entire design space by a robust and efficient two-step algorithm. In the first step, a set of univariate stable and passive macromodels is built. In the second step, a geometrical structure that determines the connections between the scattered data points in the design space is built to perform multivariate positive interpolation.**

***Index Terms*—Interpolation, parametric macromodeling, passivity, rational approximation, scattered data.**

I. Introduction

REAL-TIME design space exploration, design optimization and sensitivity analysis require the development of accurate parametric macromodels, describing the dynamic behavior of scalable systems that are characterized by time or frequency and several design variables, such as geometrical layout or substrate characteristics.

A rational parametric macromodeling method was proposed in [1] as a multivariate extension of the Orthonormal Vector Fitting (OVF) technique. It is able to accurately model highly dynamic parameterized frequency responses, but it does not guarantee stability and passivity. More recently, a technique to build parametric macromodels for S-representations that are stable and passive over the entire design space was presented in [2]. It exploits a tensor product multivariate interpolation scheme to build multivariate macromodels. However, a limitation of this method is the assumption of a fully filled, but not necessarily equidistant, rectangular grid for the data samples, which does not allow its application to scattered data grids.

This letter presents a novel technique that overcomes the restriction on the structure of the data samples present in [2] and can cope with scattered data. It is able to build accurate multivariate rational macromodels for S-representations that are stable and passive by construction over the entire design space. The technique is validated by a numerical example.

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II. Parametric Macromodeling

The goal of the proposed algorithm is to build a multivariate representation $\mathbf{R}(s,\vec{g})$ which accurately models a large set of K_{tot} scattered data samples $\{(s,\vec{g})_k, \mathbf{H}(s,\vec{g})_k\}_{k=1}^{K_{tot}}$ and guarantees stability and passivity over the entire design space $\mathcal{D}_{(\vec{g})}$. These data samples depend on the complex frequency $s = j\omega$, and several design variables $\vec{g} = (g^{(n)})_{n=1}^{N}$, such as the layout features of a circuit (e.g. lengths, widths,…) or the substrate parameters (e.g. thickness, dielectric constant, losses,…). The aim of this letter is to extend the technique presented in [2] to deal with scattered distributed data samples.

A. Root Macromodels

Starting from a set of data samples $\{(s,\vec{g})_k, \mathbf{H}(s,\vec{g})_k\}_{k=1}^{K_{tot}}$ a frequency dependent rational model in a pole-residue form is built for all grid points in the design space by means of the Vector Fitting (VF) technique [3]. A pole-flipping scheme is used to enforce strict stability [3] and passivity enforcement can be accomplished using one of the robust standard techniques [4]–[6]. The result of this initial step is a set of rational univariate macromodels, stable and passive, called *root macromodels*. This initial step allows the separation of frequency from the other parameters, in other words frequency is treated as a special parameter. Every *root macromodel* is related to a certain point in the design space. The construction of the *root macromodels* results in a family of univariate rational models related to a certain set of points in the design space. The authors consider the design space $\mathcal{D}_{(\vec{g})}$ as the parameter space $\mathcal{P}_{(s,\vec{g})}$ without frequency. $\mathcal{P}_{(s,\vec{g})}$ contains all parameters $(s,\vec{g})$. If the parameter space is N-dimensional, the design space is (N-1)-dimensional.

B. N-D Macromodeling

Once a set of *root macromodels* is available, the next step of the algorithm is focused on gluing together the *root macromodels* by a multivariate interpolation scheme to obtain a parametric macromodel $\mathbf{R}(s,\vec{g})$. This multivariate representation models the set of K_{tot} data samples $\{(s,\vec{g})_k, \mathbf{H}(s,\vec{g})_k\}_{k=1}^{K_{tot}}$, while preserving stability and passivity over the entire design space. Before performing the interpolation process, the design space is divided into cells using simplices [7]. In 2-D this process is called triangulation, in higher dimensions one speaks of a tessellation. A simplex, or N-simplex, is the N-D analogue of a triangle in 2-D and a tetrahedron in 3-D. For each data distribution many tessellations can be constructed. The minimal requirement is that the simplices do not overlap, and that there are no holes. Delaunay tessellation [8] is a well-know tessellation technique stemming from computational geometry. It is used in different fields such as mesh generation, surface reconstruction, molecular modeling and tessellation of solid shapes. Delaunay tessellation in an N-dimensional space is a space-filling aggregate of simplices and can be performed using standard algorithms [9]. We indicate a simplex region of the design space as $\Omega_i, i = 1, \ldots, P$ and the corresponding $\mathrm{N}+1$ vertices as $\vec{g}_k^{\Omega_i}$, $k = 1, \ldots, N+1$. A simplex in N dimensions has $\mathrm{N}+1$ vertices. Once the tessellation of the design space is accomplished, a tessellation-based linear interpolation (TLI) is used to build a parametric macromodel. TLI performs a linear interpolation inside a simplex with arbitrary vertices using barycentric coordinates [10] and it is therefore a local method. If the N-dimensional volume of the simplex does not vanish, i.e. it is non-degenerate, any point enclosed by a simplex can be expressed uniquely as a linear combination of the $\mathrm{N}+1$ simplex vertices. A multivariate macromodel can be written as

$$\mathbf{R}(s, \vec{g}) = \sum_{k=1}^{N+1} \mathbf{R}\left(s, \vec{g}_k^{\Omega_i}\right) \ell_k^{\Omega_i}(\vec{g}) \tag{1}$$

where Ω_i is the simplex the contains the point $\vec{g}$ and the barycentric coordinates $\ell_k^{\Omega_i}(\vec{g})$ satisfy the following properties:

$$0 \leq \ell_k^{\Omega_i}(\vec{g}) \leq 1 \tag{2}$$

$$\ell_k^{\Omega_i}\left(\vec{g}_i^{\Omega_i}\right) = \delta_{k,i} \tag{3}$$

$$\sum_{k=1}^{N+1} \ell_k^{\Omega_i}(\vec{g}) = 1. \tag{4}$$

We remark that the interpolation process is local, because the multivariate model $\mathbf{R}(s, \vec{g})$ in a certain point $\vec{g} = (g^{(1)}, \ldots, g^{(N)})$ only depends on the $\mathrm{N}+1$ *root macromodels* at the vertices of the simplex that contains the point $\vec{g}$. The TLI interpolation method belongs to the general class of positive interpolation schemes [11]. In the bivariate (s, g) case the interpolation scheme boils down to piecewise linear interpolation [2]. Stability is automatically preserved in (1), as it is a weighted sum of strictly stable rational macromodels. The proof of the passivity preserving property of the proposed technique over the entire design space is given in Section II-C.

C. Passivity Preserving Interpolation

A proof similar to [2] is provided for the passivity preserving property of the new macromodeling technique. A linear network described by S-matrix $\mathbf{S}(s)$ is passive if [12]:

1) $\mathbf{S}(s^*) = \mathbf{S}^*(s)$ for all s, where "$*$" is the complex conjugate operator.
2) $\mathbf{S}(s)$ is analytic in $\Re e(s) > 0$.
3) $\mathbf{I} - \mathbf{S}^t(s^*)\mathbf{S}(s) \geq 0; \forall s : \Re e(s) > 0$.

Concerning the *root macromodels*, conditions 1) and 2) are always satisfied since all complex poles/residues are always considered along with their conjugates and strict stability is imposed by pole-flipping. Condition 1) is preserved in the proposed multivariate representation (1), as it is a weighted sum with real nonnegative weights of systems respecting this first condition. Condition 2) is preserved in (1), as it is a weighted sum of strictly stable rational macromodels. Condition 3) is enforced, if needed, on the *root macromodels* by using a standard passivity enforcement technique [4]–[6]. It is equivalent to the condition $\|\mathbf{R}(s)\|_\infty \leq 1$ ($\mathbf{H}_\infty$ norm) [13], i.e., the largest singular value of $\mathbf{R}(s)$ does not exceed one in the right-half plane. Using this equivalent condition, we can write

$$\|\mathbf{R}(s, \vec{g})\|_\infty \leq \sum_{k=1}^{N+1} \left\|\mathbf{R}\left(s, \vec{g}_k^{\Omega_i}\right)\right\|_\infty \ell_k^{\Omega_i}(\vec{g}) \leq \sum_{k=1}^{N+1} \ell_k^{\Omega_i}(\vec{g}) = 1. \tag{5}$$

Condition 3) is satisfied by construction in (1). We have demonstrated that all three passivity conditions for S-representations are preserved in the novel parametric macromodeling algorithm, using the sufficient conditions (2)–(4) related to the interpolation kernels.

Fig. 1. Geometry of the double folded stub microstrip bandstop filter.

III. Numerical Example

A. Double Folded Stub Microstrip Bandstop Filter

The present technique is used to model a double folded stub microstrip bandstop filter [2] that is shown in Fig. 1. The substrate is 0.1270 mm thick with a relative dielectric constant $\epsilon_r = 9.9$ and a loss tangent $tan\ \delta = 0.003$. The S-matrix is modeled as function of the varying length of each folded segment $L \in [2.08 - 2.28]$ mm and varying spacing between a folded stub and the main line $S \in [0.091 - 0.171]$ mm over the frequency range [5–20] GHz. All data is simulated by ADS-Momentum.[1]

403 *root macromodels* are built at 403 scattered points in the design space by means of VF. These scattered points in the 2-D design space composed of the variables (L, S) are chosen by a maxmin Latin hypercube design [14], enhanced by adding some data points on the boundary of the design space. The required number of poles in VF is adaptively selected for each *root macromodel* using a bottom-up approach, in such a way that the corresponding maximum absolute model error for each entry of the scattering matrix is smaller than -60 dB. The passivity of each *root macromodel* is verified by checking the eigenvalues of the Hamiltonian matrix [13] and it is enforced if needed. Once a triangulation of the 2-D design space is performed, a trivariate macromodel is obtained using the TLI technique. A passivity test on a dense sweep over the design space has confirmed the theoretical claim of overall passivity. Once the parametric macromodel is built, it is validated over a reference grid of $300 \times 60 \times 60$ samples $(freq, L, S)$. Fig. 2 shows all data points in the design space selected to build ($\circ$) and validate ($\cdot$) the parametric macromodel. Fig. 3 shows the magnitude of the parametric macromodels of $\mathbf{S}_{11}(s, L, S)$ and $\mathbf{S}_{21}(s, L, S)$ for the length values $L = \{2.08, 2.28\}$ mm. Fig. 4 shows the distribution of the absolute error over the dense reference grid in a histogram. The maximum absolute error over the reference grid

[1] *Momentum EEsof EDA*, Agilent Technologies, Santa Rosa, CA.

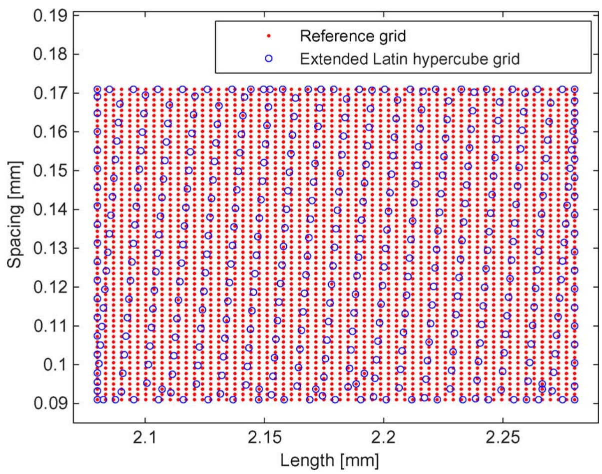

Fig. 2. Data points in the design space to build $(\circ)$ and validate $(\cdot)$ the parametric macromodel.

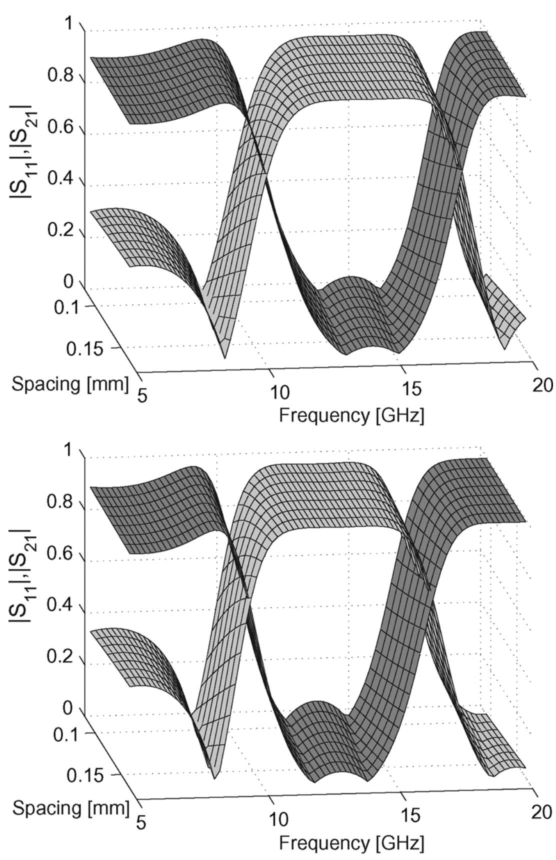

Fig. 3. Magnitude of the trivariate models of S_{11} (light grey surface) and S_{21} (dark grey surface) for $L = 2.08$ mm (top) and $L = 2.28$ mm (bottom).

is bounded by -60.5 dB. The parametric macromodels describe the behavior of the system very accurately, while guaranteeing overall stability and passivity.

IV. Conclusion

We have presented a new parametric macromodeling technique for scattered S-parameter data samples. A two-step algorithm is used: first a family of stable and passive *root macromodels* is built for a set of data points in the design space and then a tessellation-based linear interpolation scheme is used to provide a parametric macromodel. A numerical example confirms the capability of the algorithm to provide accurate parametric macromodels of dynamic systems starting from scattered data samples, while guaranteeing stability and passivity over the complete design space.

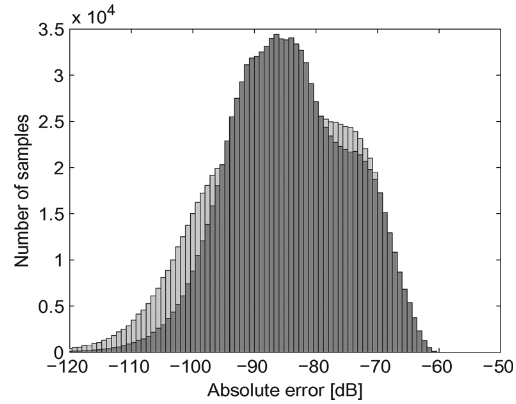

Fig. 4. Histogram: error distributions of the trivariate models of S_{11} (light grey) and S_{21} (dark grey) over 1080000 validation samples.

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