A Power-Scalable Linearized Model for RF Power Amplifiers Starting from S-parameter Measurements

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***Abstract* - A method is proposed to estimate the 'best linear approximation' of RF power amplifiers excited by spectrally rich signals under variable input power levels. Since the input-output behavior of these amplifiers is not only a function of the frequency, but also of the input power, a 2-dimensional model that linearizes the behavior will be needed. The proposed method estimates a 2D rational model in the frequency variable $j\omega$ and the input power P. The proposed approach is suitable for the experimental characterization of existing RF power amplifiers.**

***Keywords* - Nonlinear, black-box, Maximum Likelihood Estimation, Best Linear Approximation, RF amplifiers, RF measurements.**

I. Introduction

In order to design performant telecommunication systems, system designers rely heavily on models of the utilized RF components to predict the system response. Physical (white box) models, that describe the true behavior of the RF component based on the knowledge of their internal structure, can be used to yield such models. However, the internal structure of the components is (usually) not disclosed by the RF component manufacturers. For this reason, black box behavioral models that describe the input-output behavior of the system are an alternative solution. These models can be made easy to evaluate and do not contain structural or component information.

RF components are often considered to be Linear Time Invariant (LTI) systems and, hence, all the known tools of the linear system identification can be used to model these devices. However, the constant pressure of low-power and high bandwidth applications pushes an increasing number of devices beyond the edges of their linear operating range into the nonlinear operation region.

Instead of giving up the linear framework totally, one can still use the LTI system model as an approximation of the device operation characteristic under a given excitation signal class. Hence, the nonlinear behavior will be linearized around an input signal class. Changing the considered input signal class changes the operation condition and leads to a different linear approximation of the device under test (DUT). The price to pay for this approximate black-box approach is that the model depends on the power of the applied input signal. As a consequence, one will no longer generate a single LTI model for a device, but will rather provide one dedicated model for each class of excitation signals.

In the BLA, the input-output behavior of a DUT is approximated by its related linear dynamic system (RLDS) and a nonlinear noise source with a frequency dependent variance [1]. The BLA is in essence a frequency response function, which is valid for a class of excitation signals that have their power spectrum (frequency band, power level and power density function (pdf)) in common [2]. Here, a multisine signal will be used to set the operating point of the nonlinearity of the DUT and hence the BLA will be valid for all the signals that have the same power spectrum. Hence this approach will be valid for modulated signals with a big carrier and small sidelobes (eg. AM signals). For this class of considered excitation signals, it can be proven that the BLA is the "best" approximation in the sense that it minimizes the least squares difference between the output of the linear approximation and the actual system response.

The BLA has been defined for a class of excitation signals that have a common power spectrum. Hence, the BLA is only valid for one input power level. As a consequence, whenever the input power changes, another BLA will be found, as was already shown in [3]. Since telecommunication system designers want to optimize their system, they need to know how the components will react when the input power changes.

This problem can be solved by extending the BLA towards a 2D model, such that the model not only shows the frequency dependency but also the dependency on the power of the excitation signal.

Note that the input power level is not the only factor that can change the BLA. Assuming that the input power is fixed, a change in the input or output impedance of the amplifier can also lead to a change in the BLA if this change is large enough to modify the operating point of the nonlinearity significantly. This motivates the need for a multi-dimensional approximate model.

A. Existing methods

De Locht et al. [4] already proposed a method for the BLA to cope with a varying input power. To do so, the parametric BLA is estimated for each power level in a set of input power levels. Afterwards, the corresponding estimated poles and zeros between the different power levels are grouped together. Hence, the different locations of each pole/zero over the input power level are interpolated, and as a result, the pole and zero trajectories are found and used to build up a metamodel (a model of models). Hence this metamodel copes with the varying input power. This method introduces two possible sources of model errors: the errors due to the estimation of the poles and zeros of the BLA for each considered input power level and the errors due to the interpolation of the poles and zeros over the power dimension. Furthermore, one can wonder how this method could and should be expanded towards a multi-dimensional approach.

Hendrickx et al. [5] proposed a surrogate modelling approach. The Surrogate Modelling Toolbox (SUMO) [6] gave numerical problems during the fitting of a 2D rational model. A 2D neural network model based on radial basis functions could indeed be found by using a genetic algorithm, but the model obtained is no longer a rational model. Unfortunately, these models have a lot of parameters and hence are difficult to understand and do not give much physical insight. Furthermore, one has to note that the proposed approach is a fitting procedure instead of an estimation. Fitting methods do not take into account the noise information and hence it is impossible to get a measure of the quality of the obtained model.

B. Proposed method

In this paper, a 2D rational (parametric) model $G(j\omega, P, \theta)$, that is valid for every frequency f and input power P within a considered frequency and input power range, will be identified:

$$G(j\omega, P, \theta) = \frac{\sum_{(m_b, n_b) = (0,0)}^{(M_b, N_b)} b_{m_b, n_b} (j\omega)^{m_b} P^{n_b}}{\sum_{(m_a, n_a) = (0,0)}^{(M_a, N_a)} a_{m_a, n_a} (j\omega)^{m_a} P^{n_a}} \quad (1)$$

Here (M_b, N_b) and (M_a, N_a) are respectively the orders of the numerator and the denominator and $\theta = [a_{m_a, n_a}, b_{m_a, n_a}]$ are the parameters of this model.

The parameters of this model will be estimated by using a maximum likelihood estimator (MLE). The reason for this choice of estimator are its properties: consistency, asymptotic normality and asymptotic efficiency [2]. The first property guarantees that most of the probability mass gets more and more unimodally concentrated around its limiting value. Asymptotic normality means that the distribution of the estimated parameters tends to a Gaussian distribution. The importance of the assymptotic normality is that the MLE can be used to calculate uncertainty bounds on the estimates with a given confidence level. The assymptotic efficiency expresses that the MLE reaches the Cramér-Rao lower bound. This means that, asymptotically, no unbiased estimator has a lower mean squared error than the MLE. By using the maximum likelihood estimator, a 2D rational model for the amplifier will be obtained instead of a model of models (metamodel). Due to the statistical properties of the MLE, a measure for the quality of the obtained model can be found by considering the model error and the uncertainty bounds on the estimates.

However, this estimator is only able to reach the global minimum when "good" starting values for the parameters are given. For this reason, a method is used that combines a good global minimization procedure with the good statistical properties of the MLE [2]. To do so, one starts by estimating the starting values with a Weighted Generalized Total Least Squares (WGTLS) [2]. Next, these starting values are used for a Bootstrapped Total Least Squares estimation (BTLS) [2]. This estimation procedure provides the starting values for the final estimation: a Maximum Likelihood Estimation (MLE). Furthermore, one has to notice that an output error framework is assumed since one wants to estimate a 2D parametric model that is closely related to the BLA [2].

II. From Measurements to 2D Rational Model

A. The measurement setup

In this particular example, the dynamic non-linearity of a microwave amplifier will be modeled as a function of both the frequency and the power of the excitation signal. The difference between the full blown BLA measurement setup [7] and the setup used here and shown in Fig. 1 is that the large excitation signal that is used to set the operating point of the non-linearity is a sinewave rather than a multisine signal. This choice has been imposed by limitations of the current measurement setup. The measurement of the frequency response of the DUT is obtained by injecting a small probing sinewave signal through port 1 of the Performance Network Analyzer (PNA) (see Fig. 1). The power level of this signal is to be selected such that its influence on the operating point of the nonlinear device is negligible. This hypothesis can easily be verified: the measured frequency response that is obtained at 2 small, but different probing levels for the same large signal amplitude should be equal up to the uncertainty of the measurements. This has been checked for the power levels that were applied during the experiments.

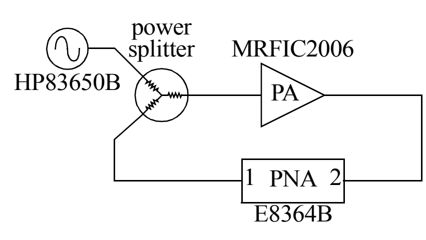


Fig. 1. Scheme of the measurement setup

B. Description of the experiment

The MRFIC2006 power amplifier from Motorola [8] is used as a DUT for this experiment. The supply voltages of this amplifier are 1V for V_{cc1} that biases the first amplification stage and 4V for V_{cc2} that biases the second and output stage (see Fig. 2). Note that the supply voltages are chosen different from the application note, in order to enhance the input power dependency.

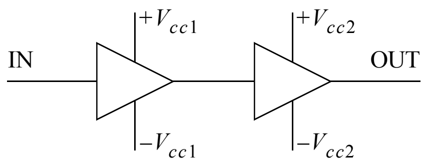


Fig. 2. Supply voltages of the Motorola MRFIC2006

The amplifier has been excited by a large sinewave signal with a frequency of 1 GHz to set the operating point of the nonlinearity. The power of this signal has been swept in order to measure the frequency response of the amplifier over a power range from -14 dBm to 4 dBm in steps of 0.6 dBm. This sinewave signal is generated by a HP83650B signal generator and fed to the amplifier through a power splitter.

An E8364B Performance Network Analyzer (PNA) was used to measure the S_{21} transfer function of the device under test. The device injects the small (-20dBm) probing signal in a frequency range from 300MHz to 1700MHz with a frequency resolution of 1MHz. Due to the rather modest frequency selectivity of the detectors of the PNA, the measured small signal response cannot be measured accurately for frequencies in the vicinity of the frequency of the large sinewave signal. For this reason, the behavior of the amplifier is not measured in a frequency band of 400MHz centered around the frequency of the pump signal. One of the advantages of the frequency domain approach is that this unequally spaced frequency grid poses little or no significant problems to the estimation procedure. Of course, if sharp resonances were to be expected in the unmeasured frequency band, under-modeling may result. In order to estimate the sample variance of the noise on the measurements, 10 repeated measurements were performed. The sample variance is then plugged in the MLE instead of the real unknown noise variance.

C. Calibration of the experiment

The proposed measurement setup has the disadvantage that the power splitter and the sine source are located in between the wave reflectometers of the PNA (see Fig. 1) and the DUT. This implies that even a minimal change in the setup of the source will automatically require a recalibration of the complete measurement setup. Instead of one single SOLT calibration [9], as much as 31 calibrations were needed to obtain correct measurements. This disadvantage can be avoided in future measurements using a different Vector Network Analyzer (VNA), where the additional source can be put outside the wave reflectometers.

D. Estimating a model out of the measurement data

By measuring the S_{21} parameter for every chosen frequency f_k and input power P_l, one gets the complex value of the transfer function. Since this linearizes the input-output behavior of the RF amplifier under the considered operation condition (i.e. a large sinewave excitation signal is used to set the operating point of the non-linearity), this gives a nonparametric model that is closely related to the BLA:

$$G_m(j\omega_k, P_l) = \frac{Y(j\omega_k, P_l)}{U(j\omega_k, P_l)} = S_{21}(j\omega_k, P_l) \quad (2)$$

This model is shown in Fig. 3 and represents the input-output behavior of the amplifier under test for the measured frequencies and input powers. Starting from this measurement data, a parametric model (1) will be estimated as is explained

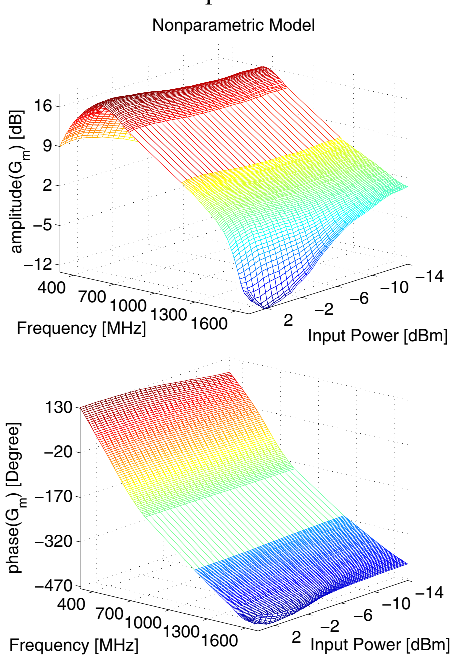


Fig. 3. Amplitude and phase of $G_m(j\omega_k, P_l) = \frac{Y(j\omega_k, P_l)}{U(j\omega_k, P_l)}$

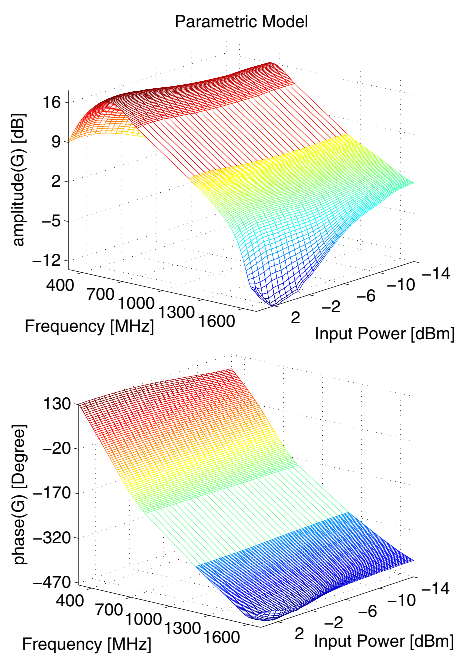
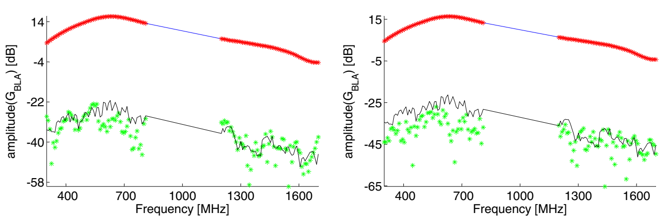


Fig. 4. Amplitude and phase of the estimated 2D rational (parametric) model $G(j\omega, P, \hat{\theta})$

in I.B. The orders (M_b, N_b) and (M_a, N_a) of this parametric model are chosen equal to 8 for the frequency dependency as well as for the input power dependency. When a lower order is chosen, the model is not capable to explain all the measured dynamics. For high orders, the estimated model will also capture the noise disturbances. Furthermore, higher orders, results in unstable models over the power and frequency range. For these reasons, an assessment has to be made between high and low order models. The model chosen is represented in Fig. 4.

E. Model Validation

Since an output error framework is assumed, the appropriate method to validate the estimated model is to calculate the residuals $r_{k,l} = \hat{G}(j\omega_k, P_l) - G_m(j\omega_k, P_l)$. Afterwards, the norm of the residuals over the frequencies is taken and compared with the 95% uncertainty level. When circular complex distributed noise is assumed, it can be proven that the 95% uncertainty level corresponds with the $\sqrt{3}\sigma$ level, where $\sigma^2(f_k, P_l)$ is the variance of the measurements for a given frequency f_k and a given input power P_l. Since also the calibration errors have to be taken into account, this estimate follows the method explained in [10]. This residual analysis gives an idea about the quality of the obtained parametric model. This is the major advantage of this procedure.



(a) (b)

Fig. 5. The red crosses represent the nonparametric model G_m, the blue solid line shows the parametric model $\hat{G}$. For case (a) the order of the numerator and denominator are chosen: $M_b = M_a = 8$ and $N_b = N_a = 8$. For case (b), the order of numerator and denominator are chosen: $M_b = M_a = 15$ and $N_b = N_a = 8$. The green crosses represent the residuals between the nonparametric and the parametric model, and the black solid line shows the $\sqrt{3}\sigma$ level. Note that input power slices of the models are considered for an input power of -6.2dBm.

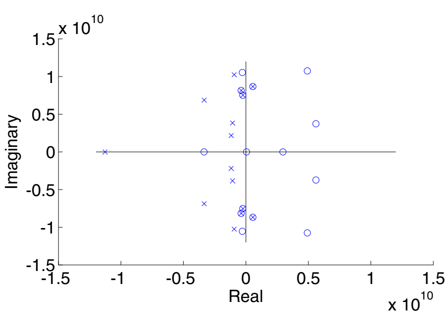


Fig. 6. The parametric model $\hat{G}$ ($M_b = M_a = 15$ and $N_b = N_a = 8$), evaluated for an input power of 3.4dBm, has been parametrized in pole (represented by a cross) and zero (represent by a circle) locations. It is clear that this model is not stable for this power slice. Furthermore one can notice that the unstable pole can be cancelled by a zero, indicating the model order can be lowered.

In Fig. 5a, the parametric model (blue solid line), the nonparametric model (red crosses), the residuals (indicated by green crosses) and the $\sqrt{3}\sigma$ level (solid black line) are represented for an input power of -6.2dBm. For the obtained model, 58% of the residuals lies beneath the $\sqrt{3}\sigma$ level. When the model order has been chosen equal to 15 for the frequency dependency and equal to 8 for the power dependency, 83% of the residuals will lie beneath the $\sqrt{3}\sigma$ level (see Fig. 5b). However, this estimated model is no longer stable. This is represented in Fig. 6 and shows that the order of numerator and denominator should be chosen well.

III. Conclusion

A parametric 2-dimensional rational model is introduced that takes into account the frequency as well as the input power dependency of a device under test. The obtained model linearizes the nonlinear behavior of the investigated power amplifier. A measure for the quality of the obtained model can be given since the noise variance has been taken into account. Hence one is not only able to estimate a 2-dimensional rational model, but one can also find a measure for the quality/ uncertainty of the obtained model.

References

[1] J. Schoukens, T. Dobrowiecki, R. Pintelon. Parametric and nonparametric identification of linear systems in the presence of nonlinear distortions - A frequency domain approach, *IEEE Transactions on Automatic Control*, vol. 43, no. 2, pp. 176-190, March 1998.

[2] R. Pintelon, J. Schoukens (2001). *System Identification. A frequency domain approach*. IEEE Press, New Jersey.

[3] Y. Rolain, W. Van Moer, R. Pintelon, J. Schoukens. Experimental Characterization of the Nonlinear Behavior of RF Amplifiers, *IEEE Transactions on Microwave Theory and Techniques*, vol. 45, no. 8, pp.3209-3218, August 2006.

[4] L. De Locht, G. Vandersteen, Y. Rolain, R. Pintelon. Estimating parameterized scalable models from the best linear approximation of nonlinear systems for accurate high-level simulations, *IEEE Transactions on Instrumentation and Measurement*, vol. 55, no. 4, pp. 1186 - 1191, Aug. 2006.

[5] W. Hendrickx, T. Dhaene. Sequential Design And Rational Metamodelling, Winter Simulation Conference Proceedings, Orlando, USA, 2005, pp. 290-298.

[6] D. Gorissen, L. De Tommasi, K. Crombecq, T. Dhaene, "Sequential Modeling of a Low Noise Amplifier with Neural Networks and Active Learning," Neural Computing and Applications (Springer), 10 pages, online, December 2008.

[7] W. Van Moer, Y. Rolain. Best Linear Approximation: revisited, accepted for the IEEE Instrumentation and Measurement Technology Conference Proceedings, Singapore, 2009.

[8] Motorola, *"The MRFIC Line 900MHz 2 Stage PA"*, Technical Datasheet, 1998.

[9] Hewlett-Packard. *"Student guide for Basic Network Measurements using the HP8510A Network Analyser System"*, HP8510A+24D, December 1986.

[10] R. Pintelon, J. Schoukens, G. Vandersteen. Nonparametric Noise Modeling in the Presence of Exogenous Inputs, accepted for the IEEE Instrumentation and Measurement Technology Conference Proceedings, Singapore, 2009.