Parameterized macromodels of multiconductor transmission lines

G. Antonini
UAq EMC Laboratory
Dept. of Electrical Engineering
University of L'Aquila
67100 Monteluco di Roio, Italy

F. Ferranti, T. Dhaene, L. Knockaert
Department of Information Technology (INTEC)
Ghent University-IBBT, Sint Pietersnieuwstraat 41,
9000 Ghent, Belgium

Abstract

We introduce a novel parametrization scheme for lossy and dispersive multiconductor transmission lines (MTLs) having a cross-section depending on geometrical and physical parameters, that is suitable to interconnect modeling. The proposed approach is based on the dyadic Green's function method for the analysis of lossy and dispersive MTLs which is parameterized by using the Multivariate Orthonormal Vector Fitting (MOVF) technique to build parametric macromodels in a rational form. Design parameters, such as substrate or geometrical layout features, in addition to frequency, can be easily handled. The rational form of the multi-port macromodel describing the MTL is a direct consequence of the MOVF technique and is especially suited to generate state-space macromodels or to be synthesized into equivalent circuits, which can be easily embedded into conventional SPICE-like solvers. A numerical example is presented providing evidence of the accuracy of the proposed approach in both frequency and time-domain.

1 Introduction

Accurate modeling of previously neglected second order effects, such as crosstalk, delay and reflection, becomes increasingly important during circuit and system simulations, as the feature size and density of integrated circuits (ICs) decrease, while the signal speeds increase. The accurate and efficient prediction of these effects is crucial for a successful design. Over the years, several different techniques have been proposed for the analysis of multiconductor transmission lines [1]- [3].

More recently, in [4], the analysis of lossy and dispersive MTLs has been carried out through a spectral approach which is based on closed-form dyadic Green's function of the 1-D wave propagation problem that is expanded in a series form of orthonormal basis functions. The rational form of each term of the dyadic Green's function allows to easily identify poles and residues of the interconnect, naturally leading to a time-domain macromodel.

At the physical design stage of interconnects, the cross-section is to be defined in terms of conductor width, height, spacing and dielectric height. Their choice has to be guided by numerical tools which permit to compute the electrical performance corresponding to each set of parameters in a fast and reliable way.

To this aim, parametric macromodeling techniques that take into account design parameters along with frequency (or time) are required as alternative to full electromagnetic solvers which are usually computationally time- and resource-demanding. Parametric macromodeling techniques for MTLs were proposed in the framework of model order reduction techniques [5]- [6]. A parametrization scheme based on the generalized method of characteristics (MoC) was presented in [7].

In this paper, a new parametric macromodeling approach is presented which is based on the Green's function method presented in [4] for lossy and dispersive MTLs. It is coupled with the Multivariate Orthonormal Vector Fitting (MOVF) technique [8] allowing to handle design geometrical and physical parameters in addition to frequency. The MOVF technique builds rational parametric macromodels starting from multivariate data samples in the parameter space and combines the use of an iterative least squares estimator and orthonormal rational functions, based on a prescribed set of poles. The multivariate model can easily be reduced to a univariate frequency-dependent model in a rational form, once a set of design parameters values is fixed. The rational model can be used to generate a time-domain state-space representation [2] or synthesized in an equivalent SPICE circuit by using standard circuit synthesis techniques [9].

The organization of the paper is as follows. A short overview of the spectral approach for MTLs is given in Section 2. Section 3 illustrates the parametric macromodeling process and its coupling to the Green's function-based technique to generate a parametric macromodel of the MTL system. Finally, numerical results and validations are presented in Section 4.

2 Dyadic Green's function of MTLs

The general solution for the voltage at abscissa z of the multiconductor transmission line due to the port currents can be obtained through the impedance matrix $\boldsymbol{Z}$ representation [1].

An alternative approach based on the dyadic Green's function for the 1-D wave propagation problem is presented in [4]. According to this methodology, the voltage at abscissa z, at complex frequency s, can be computed as:

$$\begin{aligned} \boldsymbol{V}(z,s) &= \int_0^\ell \underline{\boldsymbol{G}}_V(z,z',s)\left(-\boldsymbol{Z}_{pul}(s)\boldsymbol{I}_S(z',s)\right)dz' \\ &= \underline{\boldsymbol{G}}_V(z,0,s)\left(-\boldsymbol{Z}_{pul}(s)\boldsymbol{I}(0,s)\right) \\ &+ \underline{\boldsymbol{G}}_V(z,\ell,s)\left(-\boldsymbol{Z}_{pul}(s)\boldsymbol{I}(\ell,s)\right) \end{aligned} \quad (1)$$

where the dyadic Green's function $\underline{\boldsymbol{G}}_V(z,z',s)$ for the multiconductor transmission line problem is:

$$\underline{\boldsymbol{G}}_V(z,z',s) = -\sum_{n=0}^{\infty}\phi_n(s)A_n^2\psi_n(z)\psi_n(z'), \quad (2)$$

and

$$\phi_n(s) = \left[\boldsymbol{\gamma}^2(s) + \left(\frac{n\pi}{\ell}\right)^2 \boldsymbol{U}\right]^{-1}, \tag{3a}$$

$$\boldsymbol{\gamma}^2(s) = \boldsymbol{Z}_{pul}(s)\boldsymbol{Y}_{pul}(s), \tag{3b}$$

$$\psi_n(z) = \cos\left(\frac{n\pi}{\ell}z\right), \tag{3c}$$

$$\psi_n(z') = \cos\left(\frac{n\pi}{\ell}z'\right), \tag{3d}$$

and $A_0 = \sqrt{1/\ell}, A_n = \sqrt{2/\ell},\ n = 1, \cdots, \infty, \boldsymbol{U}$ is the unitary dyadic. Finally, the spectral representation of the $\boldsymbol{Z}$ matrix is generated as:

$$\begin{bmatrix} \boldsymbol{V}(0,s) \\ \boldsymbol{V}(\ell,s) \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{I}(0,s) \\ \boldsymbol{I}(\ell,s) \end{bmatrix} = \sum_{n=0}^{\infty} \begin{bmatrix} \boldsymbol{Z}_{n,11} & \boldsymbol{Z}_{n,12} \\ \boldsymbol{Z}_{n,21} & \boldsymbol{Z}_{n,22} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{I}(0,s) \\ \boldsymbol{I}(\ell,s) \end{bmatrix}, \tag{4}$$

where:

$$\boldsymbol{Z}_{11} = \boldsymbol{Z}_{22} = \sum_{n=0}^{\infty} \left[\boldsymbol{\gamma}^2(s) + \left(\frac{n\pi}{\ell}\right)^2 \boldsymbol{U}\right]^{-1} \cdot A_n^2 \boldsymbol{Z}_{pul}(s), \tag{5a}$$

$$\boldsymbol{Z}_{12} = \boldsymbol{Z}_{21} = \sum_{n=0}^{\infty} \left[\boldsymbol{\gamma}^2(s) + \left(\frac{n\pi}{\ell}\right)^2 \boldsymbol{U}\right]^{-1} \cdot A_n^2 \boldsymbol{Z}_{pul}(s)\cos(n\pi). \tag{5b}$$

It is composed of an infinite number of modes $\boldsymbol{Z}_n$.
Modeling frequency-dependent phenomena, such as skin effect, proximity effect and dielectric polarization losses, call for numerical computation of the per-unit-length (pul) parameters which are commonly evaluated at discrete frequency samples $s_k = j\omega_k, \omega_k = 2\pi f_k, k = 1, \cdots, K$ by means of 2-D solvers [10].

3 Parametric macromodeling of modal impedances

The spectral approach for multiconductor transmission lines allows to decompose the impedance matrix entries in modal impedances which have a rational form as shown in (5). The MOVF technique [8] is used to generate rational parametric macromodels of such modes. For ease of notation, we consider only bivariate systems in this section, but of course the full multivariate formulation can be derived in a similar way.

$$\boldsymbol{Z}_n(s,g) \simeq \tilde{\boldsymbol{Z}}_n(s,g) = \frac{\boldsymbol{N}_{Z_n}(s,g)}{D_{Z_n}(s,g)} = \frac{\sum_{p=0}^{P_{Z_n}} \sum_{v=0}^{V_{Z_n}} \boldsymbol{c}_{pv,Z_n} \phi_p(s)\varphi_v(g)}{\sum_{p=0}^{P_{Z_n}} \sum_{v=0}^{V_{Z_n}} \tilde{c}_{pv,Z_n} \phi_p(s)\varphi_v(g)} \tag{6}$$

where s is the complex frequency variable and g is a real design variable. The maximum order of the corresponding basis functions $\phi_p(s)$ and $\varphi_v(g)$ is denoted by P_{Z_n} and V_{Z_n}. The basis functions $\phi_p(s)$ and $\varphi_v(g)$ are chosen in partial fraction form and based on a prescribed set of poles. The MOVF algorithm pursues the identification of the model coefficients $\boldsymbol{c}_{pv,Z_n}$, $\tilde{c}_{pv,Z_n}$ of numerator and denominator in (6), starting from a set of data samples $\{(s,g)_k, \boldsymbol{Z}_n(s,g)_k\}_{k=1}^{K}$. A linear approximation to this nonlinear optimization problem is obtained by using an iterative least squares estimator. In this work the MOVF technique is applied to matrices and it is assumed that the different matrix entries share the same poles, so the same denominator. The combination of these modal parametric macromodels represents the final parametric macromodel of the $\boldsymbol{Z}$ matrix.
Given a fixed set of values for the parameters, the modal parametric macromodels can be reduced to univariate frequency-dependent functions. The stability and passivity for the univariate macromodel of the $\boldsymbol{Z}$ matrix are ensured by imposing these system properties on the univariate models of the modes [4]. Finally, a state-space representation and an equivalent SPICE circuit can be synthesized for the $\boldsymbol{Z}$ matrix.

3.1 Additional weighting function An additional least-squares weighting function can be added to the parametric macromodeling algorithm, when the elements to fit have a high dynamic range. It improves the relative accuracy where the elements to fit are small in their dynamic range [11] and is chosen equal to the inverse of the element magnitude:

$$w_{H_i}(s,g)_k = |(H_i(s,g)_k)^{-1}| \tag{7}$$

for $i = 1, ..., M$, where M is the maximum number of functions fitted with common poles in the same least-squares matrix.

3.2 Mode selection The infinite sum in (5) must be truncated in order to obtain a finite rational representation of the multiconductor transmission line. A bottom-up strategy based on the check of the dominant poles of the modal impedances $\boldsymbol{Z}_n$ is followed to select the number of modes. The check of the dominant poles used in this work extends the algorithm presented in [12] by looking at the ratio between the residues and the real part of the corresponding poles in the second step of the procedure. $\boldsymbol{Z}_n$ are evaluated on the minimum, mean and maximum values of each design parameter range. If for a certain number mode n all checks prove that the dominant poles are out of a defined bandwidth $\xi\omega_{max}$ (where $\xi > 1$), the algorithm ends and the number of modes is equal to $n - 1$. The algorithm is described for the bivariate case, but the full multivariate formulation is similar.

4 Numerical example

A three conductor tranmission line (length $\ell = 15$ cm) with frequency-dependent per-unit-length parameters has been modeled. It consists of two coplanar striplines. The cross sections is shown in Fig. 1. The striplines have width $w = 79\ \mu$m and thickness $t = 5\ \mu$m. The distance from the striplines to the top plane is $h_1 = 60\ \mu$m and to the bottom plane is $h_2 = 138\ \mu$m. The spacing S between the striplines is considered as a design parameter in addition to frequency. The dielectric is characterized by a dispersive and lossy permittivity which has been modeled by the wideband Debye model [13]. The infinite series in (5) has been truncated to $n_{modes} = 36$. The ranges of frequency and spacing are $f \in [1 \cdot 10^5 - 15 \cdot 10^9]$ Hz and $S \in [100 - 500]\ \mu$m, respectively.

Input: Data $\boldsymbol{Z}_{pul}(s, g_i)$, $\boldsymbol{Y}_{pul}(s, g_i)$, $g_i = \{g_{min}, g_{mean}, g_{max}\}$
Output: Number of modes n_{modes}

$convergence = false$;
$\xi > 1, \;\; 0 < \zeta < 1$;
$n = 0$;

while $convergence = false$ **do**
  %Pole check
  foreach g_i **do**
    $\boldsymbol{Z}_n = mode\_computation(\boldsymbol{Z}_{pul}(s, g_i), \boldsymbol{Y}_{pul}(s, g_i), n)$ $(eq.\ 5)$;
    $[poles_n, residues_n] = model(\boldsymbol{Z}_n)$ $(by\ Vector\ Fitting)$;
    foreach $pole_n$ **do**
      foreach $residue_n$ **do**
        if $|Im(pole_n)| < \xi\omega_{max} \cap |residue_n / Re(pole_n)| > \zeta max(|residues_n / Re(poles_n)|)$ **then**
          $n = n + 1$;
          go to foreach g_i **do**
        end
      end
    end
  end
  $convergence = true$
end

$n_{modes} = n - 1$.

Algorithm 1: *Mode selection.*

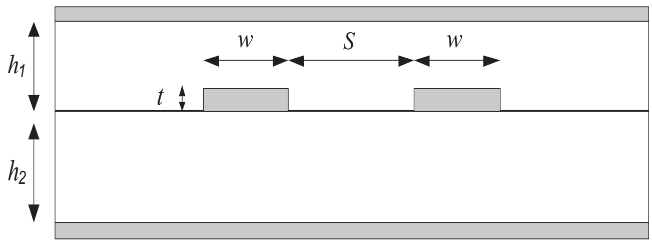

Figure 1: Cross section of the two coplanar striplines.

All 36 modal impedances have been computed over a reference grid of 250 $\times$ 40 samples, for frequency and spacing respectively. We have only used 50 $\times$ 15 samples of the previous grid and 8 and 4 poles for frequency and length, to model all modes. The maximum $RMS_{weighted}$ error of the parametric macromodels over the reference grid is equal to $4 \cdot 10^{-5}$. The magnitude of the parametric macromodel of $Z_{n,(12)}$ is shown in Fig. 2 for modes $n = \{0, 10, 24\}$. Next, these macromodels have been reduced to univariate frequency-dependent functions for the set of spacing values $S = \{193, 295, 397\}\mu$m. These points have not been used for the generation of the macromodels. The magnitude of $Z_{n,(12)}$ and its respective macromodels are shown in Fig. 3 for modes $n = \{0, 10, 24\}$, $S = 295$ μm. The results show the high accuracy of the parametric macromodeling strategy in frequency domain.
The macromodel of the $\boldsymbol{Z}$ matrix, composed of the sum of 36 modes, can be built by using the macromodels of the modal impedances. The $RMS_{weighted}$ error between the macromodel of the $\boldsymbol{Z}$ matrix and its computation from the exact transmission line theory (TLT) [1] over the reference grid is equal to $6 \cdot 10^{-4}$.

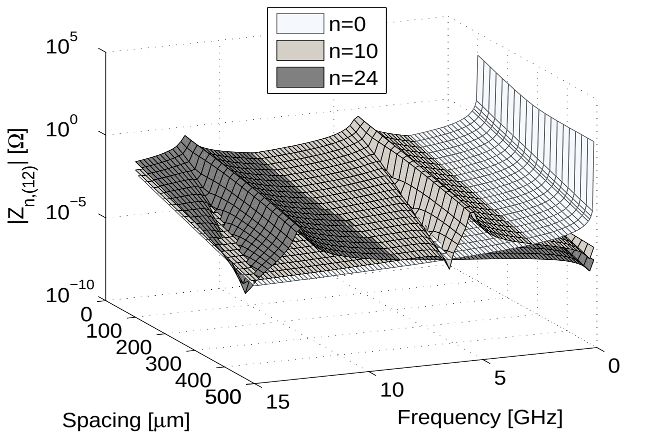

Figure 2: Magnitude of the parametric macromodel of modal impedance $Z_{n,(12)}$ (modes $n = \{0, 10, 24\}$).

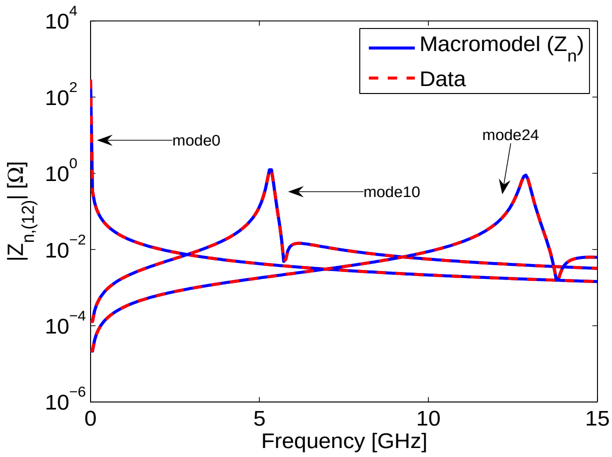

Figure 3: Magnitude of $Z_{n,(12)}$ (modes $n = \{0, 10, 24\}$, $S = 295$ μm).

As next step, we have performed some time-domain simulations to verify that the accuracy is preserved in time-domain. One line has been excited by an impulsive voltage source with amplitude 2 V, rise/fall times $\tau_r = \tau_f = 300$ ps and width 2 ns. The victim line have been terminated on the near and far-end by $R_{NE} = 50$ Ω and $R_{FE} = 50$ Ω, $C_{FE} = 1$ pF, while the driven line has been terminated on a driver and load impedance equal to $R_S = 50$ Ω and $R_L = 50$ Ω, $C_L = 1$ pF. The port voltages have been computed using the exact transmission line theory (TLT-IFFT) and a state-space realization of the frequency domain macromodel of the $\boldsymbol{Z}$ matrix. Some time domain results are shown in Fig. 4 for the set of spacing values $S = \{193, 295, 397\}\mu$m.

The port voltages results in time-domain confirm that the parametric macromodeling algorithm captures and describes second order phenomena, as the crosstalk, accurately.

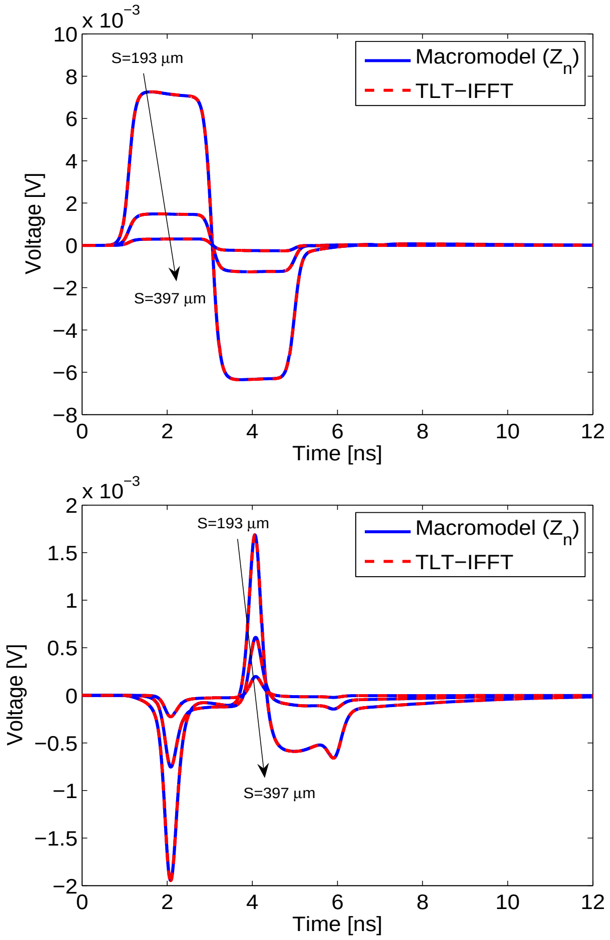

Figure 4: Input and output voltages of the victim line terminated on $R_{NE} = 50\ \Omega$ and $R_{FE} = 50\ \Omega$, $C_{FE} = 1$ pF ($S = \{193, 295, 397\}\mu$m).

5 Conclusions

We have presented an innovative parametric macromodeling technique for lossy and dispersive multiconductor transmission lines. It is able to build accurate rational macromodels with respect to physical and geometrical parameters. The use of the spectral decomposition of the impedance matrix $\boldsymbol{Z}$ leads a significant simplification of the identification process. The presented technique has been validated by a numerical example, that provides evidence of the accuracy of the proposed approach in both frequency and time-domain.

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