Modified Half-Size Test Matrix for Robust Passivity Assessment of S-Parameter Macromodels

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***Abstract*—This letter presents a modified algebraic passivity test to check if an S-parameter based macromodel with symmetric scattering matrix is passive or not. To construct the passivity test matrix, the inversion of another matrix is required that can possibly be singular. If this is the case, such methods will fail since they cannot be applied directly. This letter proposes an elegant solution to deal with this problem. The effectiveness of the approach is illustrated by two numerical examples.**

***Index Terms*—Hamiltonian matrix, macromodeling, passivity test, S-parameters, transfer matrix, vector fitting.**

I. Introduction

The characterization of broadband linear systems from measured or simulated frequency responses by Vector Fitting has received a lot of attention in literature [1], [2]. Although stability of the macromodel can be enforced by a simple pole-flipping scheme, the passivity is not guaranteed by construction. Nevertheless, passivity of the macromodel is of crucial importance since a non-passive macromodel may lead to unstable transient simulations in an unpredictable manner. Fortunately some algebraic passivity tests can be used to verify if the macromodel is passive or not and to pinpoint the exact boundaries of possible passivity violations. Such passivity tests are often based on the existence of purely imaginary eigenvalues of an associated Hamiltonian matrix [3].

In the case of systems with a symmetric scattering matrix, a half-size test matrix can be derived that is only half the size of the Hamiltonian matrix. It is shown in [4] that this leads to significant savings in terms of computation time and memory resources. Unfortunately, this method may fail in some cases, since the construction of the half-size test matrix requires the inversion of another matrix that is possibly singular. A practical solution to deal with this problem is to perturb the feedthrough matrix of the state-space model by a small amount. Although this approach can sometimes be effective, a more elegant solution is proposed in this letter. Two numerical examples illustrate that the technique is accurate and reliable.

Once the regions of the passivity violations are identified, standard passivity enforcement techniques can be applied for compensation. Such methods perturb the parameters of the state-space model (such as e.g., the poles [5], [6], residues [7], [8], residue eigenvalues [9], etc.) until all the passivity violations are collapsed and a passive macromodel is obtained.

A similar approach is possible for Y-parameter models [10].

II. Macromodeling

A direct application of Vector Fitting to the frequency samples yields a rational macromodel in the pole-residue form

$$S_{mn}(j\omega) = \sum_{p=1}^{P} \frac{c_p^{mn}}{j\omega - a_p^{mn}} + d^{mn} \tag{1}$$

provided that $S_{mn}(j\omega)$ represents the corresponding element on row m and column n of the scattering matrix. The poles a_p^{mn} and residues c_p^{mn} are real or come in complex conjugate pairs, while d^{mn} is a constant real term. All elements of the scattering matrix can be fitted with a common set of poles $\left(a_p^{mn} = a_p\right)$ or a separate set of poles for each scattering element. Stability of the poles can be ensured by a pole-flipping scheme, but passivity of the macromodel is not guaranteed by construction. A real state-space realization of the compound system can easily be derived as shown in [1]

$$j\omega X(j\omega) = AX(j\omega) + BU(j\omega) \tag{2}$$

$$Y(j\omega) = CX(j\omega) + DU(j\omega). \tag{3}$$

III. Passivity Condition Check

The frequency-domain definition of passivity for S-parameter based macromodels stipulates that all singular values $\sigma(j\omega)$ of scattering matrix $S(j\omega)$ are unitary bounded [12]

$$\max_{\omega} \sigma(S(j\omega)) \leq 1 \quad \forall\, \omega. \tag{4}$$

This condition can easily be verified algebraically by computing the eigenvalues of an associated Hamiltonian matrix [3]

$$M_H = \begin{bmatrix} A - BR^{-1}D^TC & -BR^{-1}B^T \\ C^TQ^{-1}C & -A^T + C^TDR^{-1}B^T \end{bmatrix} \tag{5}$$

where $R = D^TD - I$ and $Q = DD^T - I$. In the case of symmetric systems, passivity condition (4) can also be verified by solving the eigenvalues $\lambda(M_P)$ of a half-size test matrix

$$M_P = (A - B(D-I)^{-1}C)(A - B(D+I)^{-1}C). \tag{6}$$

It was shown in [4] that the smaller passivity matrix M_P gives, via the subset of its negative-real eigenvalues $-\omega^2$, the frequencies $\pm j\omega$ where a singular value of $S(j\omega)$ crosses unity. By computing the slopes of the singular value curves at the corresponding frequencies, it is possible to pinpoint the exact boundaries of a passivity violation [7]. The fast passivity test (6) can only be applied if $D - I$ and $D + I$ are non-singular.

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IV. Properties of Reciprocal Systems

Theorem 1: Let (A, B, C, D) be the state-space realization of a stable system with transfer matrix $S(j\omega)$

$$S(j\omega) = C(j\omega I - A)^{-1}B + D \tag{7}$$

then the transfer matrix of the reciprocal system $S(1/j\omega)$ with the corresponding realization $(\bar{A}, \bar{B}, \bar{C}, \bar{D})$ is given by [11]

$$S\left(\frac{1}{j\omega}\right) = \bar{C}(j\omega I - \bar{A})^{-1}\bar{B} + \bar{D} \tag{8}$$

provided that the matrices are defined by the bijective mapping

$$\begin{aligned} \bar{A} &= A^{-1}, \quad \bar{B} = -A^{-1}B, \\ \bar{C} &= CA^{-1}, \quad \bar{D} = D - CA^{-1}B. \end{aligned} \tag{9}$$

It is observed that the locus $\zeta = \cup\sigma(j\omega)$ of the singular values $\sigma(j\omega)$ of $S(j\omega)$ for all $\omega \in [-\infty, +\infty]$, and the locus $\tilde{\zeta} = \cup\sigma(1/j\omega)$ of the singular values $\sigma(1/j\omega)$ of $S(1/j\omega)$ for all $\omega \in [-\infty, +\infty]$ coincide. This is evident, since the imaginary $j\omega$ axis is mapped onto itself by taking its inverse. Therefore, the singular values $\sigma(j\omega)$ of $S(j\omega)$ are unitary bounded if this also holds for the singular values $\sigma(1/j\omega)$ of $S(1/j\omega)$.

V. Modified Passivity Check

Based on the previous considerations, it becomes clear that $S(j\omega)$ is unitary bounded if this is also true for the reciprocal system $S(1/j\omega)$ [10]. This leads to the following theorem.

Theorem 2: Let A be a stable state matrix and assume that $D - I$ or $D + I$ is singular. Then $\{1\} \in \sigma(j\omega_k)$ with $j\omega_k = \pm j/\alpha_k$ if and only if $\sigma(S(0) - I) > 0$, $\sigma(S(0) + I) > 0$, $-\alpha_k^2 \in \lambda(\bar{M}_P)$ and $-\alpha_k^2 \in \mathbb{R}^- \backslash \{0\}$ with passivity matrix

$$\bar{M}_P = (\bar{A} - \bar{B}(\bar{D} - I)^{-1}\bar{C})(\bar{A} - \bar{B}(\bar{D} + I)^{-1}\bar{C}) \tag{10}$$

provided that $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D} = S(0)$ are defined as in (9).

If $S(0) - I$ or $S(0) + I$ are also singular, then the half-size test matrix cannot be applied. In such case, the crossings can still be identified from the Hamiltonian matrix of the reciprocal system by shifting the frequency axis towards a frequency $j\tilde{\omega}$ with $S(j\tilde{\omega}) + I$ and $S(j\tilde{\omega}) - I$ non-singular, see [10] for details. It is clear from Theorem 2 that the crossings of the singular value curves $\sigma(1/j\omega)$ of $S(1/j\omega)$ are located at the imaginary frequencies that correspond to the square root of the non-zero negative-real eigenvalues $1/j\omega_k = \sqrt{-\alpha_k^2} = \pm j\alpha_k$ (see [4]). Therefore, it follows that the singular value curves $\sigma(j\omega)$ of $S(j\omega)$ will exceed unity at the frequencies $j\omega_k = \pm j/\alpha_k$.

VI. Example: Microwave RLC Filter

The proposed algorithm is applied to check the passivity of a non-passive macromodel of a 2-port RLC Filter, reported from [4]. The real state-space realization of the model is given below, so the reader can easily verify the computations

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -5 & 6 & 0 & 0 & 0 \\ 0 & -6 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 6 \\ 0 & 0 & 0 & 0 & -6 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

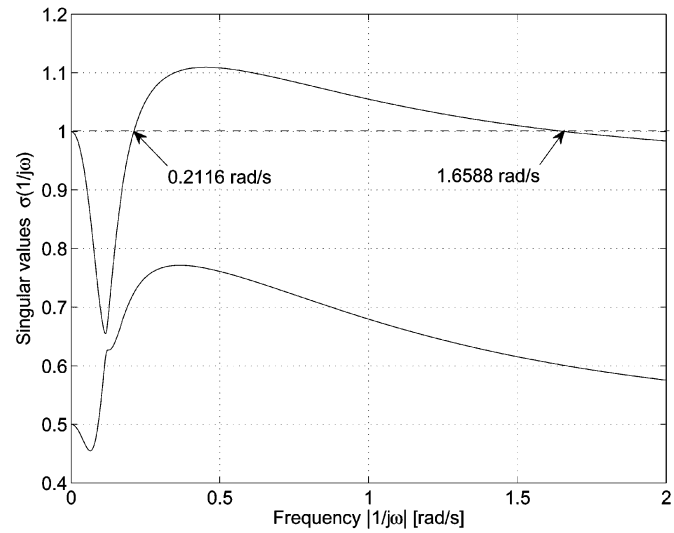

Fig. 1. Singular values of $S(1/j\omega)$ and crossover frequencies using $\bar{M}_P$.

$$C = \begin{bmatrix} 0.3 & 4 & 5 & 0.1 & 2 & 3 \\ 0.1 & 2 & 3 & 0.4 & 3 & 4 \end{bmatrix}, \quad D = \begin{bmatrix} -0.9 & 0.2 \\ 0.2 & -0.6 \end{bmatrix}.$$

It is found that the standard half-size passivity test cannot be applied, because the construction of M_P in (6) requires the inversion of $D + I$, which is a singular matrix. Therefore, the modified half size passivity test matrix $\bar{M}_P$ in (10) is formed. The eigenvalues of this matrix are shown in the left column of Table I, while the square root of the eigenvalues and its inverse are shown in the middle and the right column respectively. Based on the negative real eigenvalues $-\alpha_k^2 = \{-2.7516, -0.0448\}$ of $\bar{M}_P$, it is found that the singular values $\sigma(1/j\omega)$ of $S(1/j\omega)$ cross the unity line at $1/j\omega_k = \pm j\alpha_k = \{\pm 0.2116j, \pm 1.6588j\}$. This means that the passivity of the macromodel is violated at the frequencies $j\omega_k = \pm j/\alpha_k = \{\pm 0.6028j, \pm 4.7266j\}$ of $S(j\omega)$. To verify this result, the singular value curves of $S(1/j\omega)$ and $S(j\omega)$ are computed over the frequency ranges of interest, and the results are visualised in Figs. 1 and 2. It is clear that proposed test matrix accurately pinpoints the boundaries of the violation.

VII. Example: Quarter Wavelength Filter

The scattering matrix of a 2-port quarter wavelength filter is calculated using the planar full-wave electromagnetic simulator Agilent EEsof Momentum [13] over the frequency range of interest [1 GHz–12 GHz]. The Vector Fitting technique is then used to compute a 28-pole proper macromodel with stable poles. Fig. 3 shows the magnitude of the reflection coefficient S_{11} and the transmission coefficient S_{12}. It is found that the standard half-size passivity test M_P in (6) cannot be applied, because $D + I$ is a singular matrix. To detect possible passivity violations, the modified half-size passivity test matrix $\bar{M}_P$ in (10) is formed, and the crossings of the singular value curves are identified according to Theorem 2. Fig. 4 shows the singular value curves $\sigma(j\omega)$ of $S(j\omega)$, where the calculated crossings are marked with black dots. It is seen that several non-negligible passivity violations are accurately detected by the algorithm, most of them are located within the frequency range. Once the

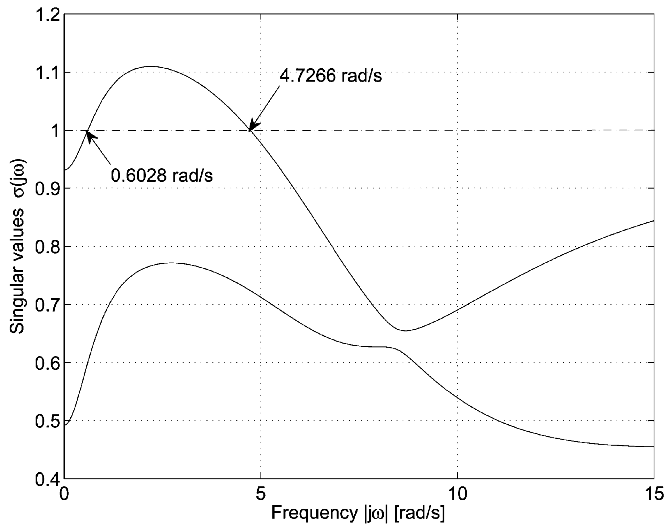

Fig. 2. Singular values of $S(j\omega)$ and crossover frequencies using $\bar{M}_P$.

TABLE I
PASSIVITY TEST MATRIX RESULTS

$-\alpha_k^2 \in \lambda(M_P)$	$1/j\omega_k = \pm j\alpha_k$	$j\omega_k = \pm j/\alpha_k$
-2.7516	$0 \quad \pm 1.6588j$	$0 \quad \pm 0.6028j$
$+0.4177$	$\pm 0.6463 \quad 0$	∓ 1.5472
-0.0448	$0 \quad \pm 0.2116j$	$0 \quad \pm 4.7266j$
$+0.0043 \quad +0.0260j$	$\pm 0.1239 \quad \pm 0.1051j$	$\mp 4.6932 \quad \pm 3.9823j$
$+0.0043 \quad -0.0260j$	$\mp 0.1239 \quad \pm 0.1051j$	$\pm 4.6932 \quad \pm 3.9823j$
0	0	∞

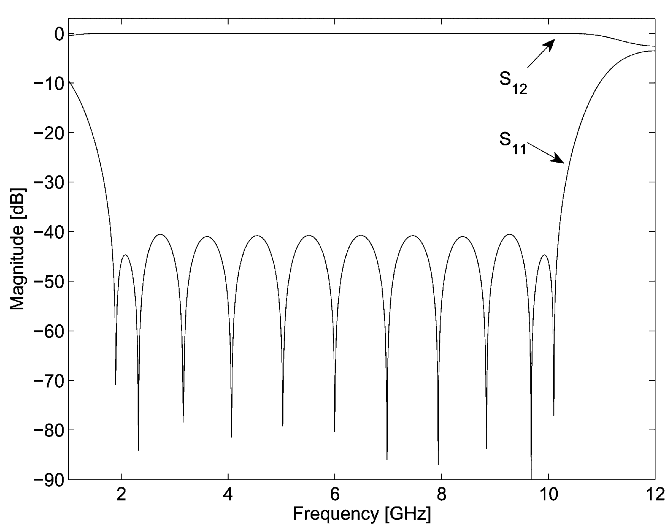

Fig. 3. Magnitude of scattering matrix elements S_{11} and S_{12}.

regions of the violations are pinpointed, standard passivity enforcement techniques such as [5]–[9] can be applied. It is also noted that the computation time to calculate the eigenvalues of $\bar{M}_P$ takes about 0.003 seconds in a MATLAB environment on an Intel Dual Core 2.4 GHz laptop computer with 2 GB of RAM memory.

VIII. CONCLUSION

This letter presents an algebraic test to assess the passivity of S-parameter based macromodels with a symmetric scattering

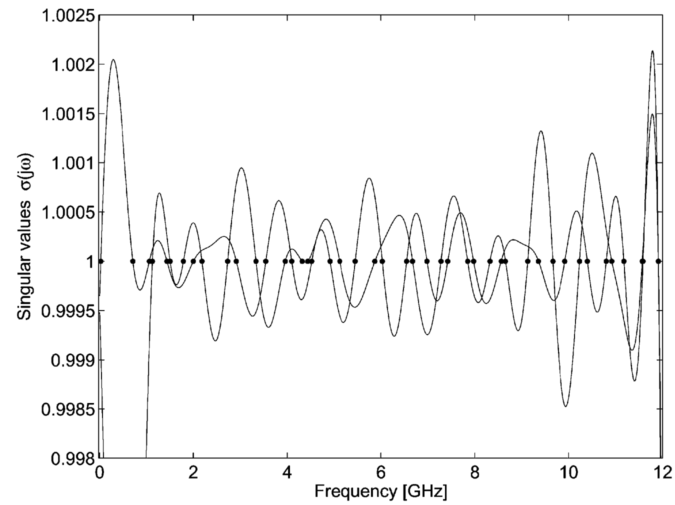

Fig. 4. Singular values of $S(j\omega)$ and crossover frequencies using $\bar{M}_P$.

matrix. It substitutes the half-size test matrix in [4] if singularity problems occur. The eigenvalues of the matrix allow accurate pinpointing of the crossover frequencies which contain the exact boundaries of all the passivity violations. In the unlikely event that both tests are inapplicable, the eigenvalues of a full-size Hamiltonian matrix should be solved.

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