

# Fast Calculation of PEEC Macromodels using Frequency Derivatives\*

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## Abstract

The accurate computation of partial elements plays a key role in ensuring the accuracy and the stability of the Partial Element Equivalent Circuit (PEEC) method. Inaccuracies in partial elements computation, even at very high frequencies, may generate late time instabilities. On the other side, accurate computation of partial elements becomes prohibitively cumbersome when performed at high frequencies. In this paper a macromodeling technique is applied, which is based on the use of frequency derivatives to allow a broadband characterization of electric and magnetic field couplings at a reduced computational cost.

## 1 Introduction

The Partial Element Equivalent Circuit (PEEC) method [1] has shown to be particularly suitable for the calculation of EMC and electrical interconnect and package (EIP) problems in combination with SPICE type circuit models since the entire problem is solved in the circuit domain. Similarly to other integral equation based techniques its time domain implementation may suffer from late time instabilities, especially when considering delays ( $(L_p, P, R, \tau)$ PEEC). They may be caused by the numerical technique used for time integration or by the discrete representation of the electromagnetic continuous problem.

Recently, the accurate computation of partial elements, describing the electric and magnetic field coupling, has been found to have an impact on the stability of PEEC circuits, besides being crucial to guarantee their accuracy [2].

As the frequency range of electrical problems grows, accurate macromodeling of Partial Element Equivalent Circuit (PEEC) models is becoming increasingly important. Fast macromodeling tools are of paramount importance to capture the frequency-dependent coupling of partial elements, in particular partial inductances ( $L_{p,mn}$ ) and coefficients of potential ( $P_{mn}$ ).

The computation of the partial inductances and coefficients of potential requires numerical integration of a frequency-dependent integral using a Gauss-Legendre quadrature [3]. The Gauss-Legendre quadrature selects integration points at the roots of the Legendre polynomial, and calculates the weights, such that the weighted sum of function evaluations is an exact integral approximation for a polynomial of specified order. In the general case, the N-point Gauss-Legendre approximation is exact for polynomials of order  $2N-1$  or less [4].

At high frequencies, the exponential term requires a large

number of integration points to resolve the integrand accurately, and therefore may require an excessive amount of computation time. It is observed that unwanted resonances occur in the partial element coupling in the extended frequency range if the order of integration is too low. To address this issue, an efficient filtering method in conjunction with a subdivision approach is presented in [5, 6], to allow the proper modeling of the magnitude and phase of the partial elements.

This paper shows that an accurate macromodel can be calculated without requiring the cumbersome computation of data samples at higher frequencies. It is found that higher-order frequency derivatives of the impedances can be computed sufficiently accurate using approximately the same order of integration as their corresponding frequency sample. By computing the lower frequency samples and their higher-order derivatives, a good approximation of the overall frequency range is obtained. This way, the computational cost can be reduced, since the expensive computation of high-frequency samples is avoided.

Traditionally, the calculation of a rational fitting model using data samples and derivatives can be done using a classical Padé approximation as described in [7]. This paper applies a generalized formulation of the Vector Fitting technique [8, 9], which includes frequency derivatives in the modeling process. This approach has the advantage that it is numerically robust and easily allows enforcement of stability, and passivity as a post-processing step [10].

## 2 Calculation of Frequency Derivatives

This section describes how the impedances  $Z_{L,mn}(s)$ ,  $Z_{C,mn}(s)$  and their frequency derivatives can be computed.

**2.1 Partial Inductances** The magnetic field coupling between volume cells  $m$  and  $n$  is described in terms of the complex partial inductance

$$L_{p,mn}(s) = \frac{\mu}{4\pi a_m a_n} \int_{v_m} \int_{v_n} \frac{e^{-s|\vec{r}_m - \vec{r}_n|/c_0}}{|\vec{r}_m - \vec{r}_n|} dv_m dv_n \quad (1)$$

where  $c_0$  is the free space speed of light,  $a_m$  and  $a_n$  the cross section of volume cells  $m$  and  $n$ . The higher-order frequency derivatives of  $L_{p,mn}(s)$  are denoted by equation (2) on top of the next page. Based on the frequency derivatives of  $L_{p,mn}(s)$ , it is easy to calculate the frequency derivatives of the complex impedance, since  $Z_{L,mn}(s) = sL_{p,mn}(s)$ . Its  $k^{th}$ -order derivatives are then denoted by

$$\frac{\delta^k(Z_{L,mn}(s))}{\delta s^k} = s \frac{\delta^k}{\delta s^k} (L_{p,mn}(s)) + k \frac{\delta^{k-1}}{\delta s^{k-1}} (L_{p,mn}(s)) \quad (3)$$

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$$\frac{\delta^k(L_{p,mn}(s))}{\delta s^k} = \frac{(-1)^k \mu}{4\pi a_m a_n c_0^k} \int_{v_m} \int_{v_n} (|\vec{r}_m - \vec{r}_n|)^{k-1} e^{-s|\vec{r}_m - \vec{r}_n|/c_0} dv_m dv_n \quad (2)$$

$$\frac{\delta^k(P_{mn}(s))}{\delta s^k} = \frac{(-1)^k}{4\pi \varepsilon S_m S_n c_0^k} \int_{S_m} \int_{S_n} (|\vec{r}_m - \vec{r}_n|)^{k-1} e^{-s|\vec{r}_m - \vec{r}_n|/c_0} dS_m dS_n \quad (5)$$

**2.2 Coefficients of potential** The electric field coupling is described by the complex coefficient of potential. Considering two cells,  $m$  and  $n$ , the mutual coefficient of potential is

$$P_{mn}(s) = \frac{1}{4\pi \varepsilon S_m S_n} \int_{S_m} \int_{S_n} \frac{e^{-s|\vec{r}_m - \vec{r}_n|/c_0}}{|\vec{r}_m - \vec{r}_n|} dS_m dS_n \quad (4)$$

where  $S_m$  and  $S_n$  represent the area of the surface cells  $m$  and  $n$ . The higher-order derivatives of  $P_{mn}(s)$  are denoted by equation (5) on top of this page. Based on the frequency derivatives of  $P_{mn}(s)$ , the frequency derivatives of the complex impedance can easily be calculated, since  $Z_{C,mn}(s) = P_{mn}(s)/s$ . Its  $k^{\text{th}}$ -order derivatives are then denoted by

$$\frac{\delta^k(Z_{C,mn}(s))}{\delta s^k} = \sum_{v=0}^k \frac{k!(-1)^v s^{-v-1}}{(k-v)!} \frac{\delta^{k-v}(P_{mn}(s))}{\delta s^{k-v}} \quad (6)$$

### 3 Macromodeling Algorithm

The goal of the Vector Fitting [9] macromodeling algorithm, is to identify the coefficients  $c_p$  and  $\tilde{c}_p$  in (7), such that the difference between the rational function approximation and all data samples ( $s, Z(s)$ ) is minimized in a least-squares sense. A linear approximation of this non-linear identification problem is obtained by solving the following set of equations ( $\tilde{c}_0 = 1$ ).

$$\sum_{p=1}^P \frac{c_p}{s - a_p} - Z(s) \sum_{p=1}^P \frac{\tilde{c}_p}{s - a_p} \approx Z(s) \quad (7)$$

In successive iterations, a Sanathanan-Koerner iteration [11] can be applied which iteratively relocates the poles of the transfer function, until a good approximation is obtained. It is found that an accurate fitting model usually results in a few iterations, provided that the initial set of starting poles is well-chosen.

In [8], it is shown that a generalized formulation can be expressed by taking the derivatives on both sides of equation (7)

$$\sum_{p=1}^P c_p \frac{\delta^k}{\delta s^k} \left( \frac{1}{s - a_p} \right) - \sum_{p=1}^P \tilde{c}_p \frac{\delta^k}{\delta s^k} \left( \frac{Z(s)}{s - a_p} \right) \approx \frac{\delta^k}{\delta s^k} (Z(s)) \quad (8)$$

provided that

$$\psi_{p,k}(s) = \frac{\delta^k}{\delta s^k} [(s - a_p)^{-1}] = (-1)^k k! (s - a_p)^{-(k+1)} \quad (9)$$

and

$$\frac{\delta^k}{\delta s^k} [Z(s) (s - a_p)^{-1}] = \sum_{v=0}^k \binom{k}{v} \psi_{p,v}(s) \frac{\delta^{k-v}}{\delta s^{k-v}} (Z(s)) \quad (10)$$

A direct application of the data samples and frequency derivatives ( $s, \delta^k/\delta s^k(Z(s))$ ) to (8) leads to a linear set of equations which can be solved in terms of the coefficients  $c_p$  and  $\tilde{c}_p$ . Based on the coefficients  $\tilde{c}_p$ , the poles of the transfer function can be computed by solving an eigenvalue problem. The prescribed poles can be replaced by the relocated poles, and the procedure is repeated iteratively until convergence is reached. Unstable poles can be flipped into the left half plane, to ensure system stability. In the final iteration, the residues of the transfer function can be calculated as a linear problem. The pole-residue form allows a directly application of passivity enforcement techniques as a post-processing step, see [10].

### 4 Example

In this example, two cells ( $\alpha$  and  $\beta$ ) are considered, with dimensions  $l_\alpha = 1.5$  mm,  $l_\beta = 1.5$  mm,  $w_\alpha = 1.5$  mm and  $w_\beta = 1.5$  mm. This ensures that the cell dimensions match the requirement  $\max(l_\alpha, l_\beta, w_\alpha, w_\beta) \leq \lambda_{\min}/20$  at the active frequency  $f_a = 10$  GHz. The maximum frequency at which the models need to be accurate is  $f_{\max} = 20 f_a = 200$  GHz. They touch each other, as shown in Figure 1. The mutual impedance  $Z_{C,mn}$  has been computed using a Gauss-Legendre integration technique, of which the order is increasingly chosen according to the frequency. The same order of integration is selected to compute the corresponding first order derivatives  $\delta Z_{C,mn}/\delta s$ .

In the first test, 5 equidistant data samples are calculated from DC up to 70 GHz and a rational fitting model is calculated using the Vector Fitting technique. The calculation of high-frequency data samples is avoided, since they require a larger amount of integration points and are therefore computationally more expensive. The RMS-error of the fitting model corresponds to  $6.3362 * 10^{-2}$ , which results in a visual distortion in the frequency response (both magnitude and phase) at the higher frequencies, as shown in Figure 2.

For comparison, the same calculations are repeated, including the first order frequency derivatives in the fitting process. It is clear that the derivatives provide additional information to the model, and therefore lead to more accurate results. As can be seen from Figure 3, the fitting model now accurately captures the frequency response of the data (both magnitude and phase), and a resulting RMS error of  $2.4734 * 10^{-4}$  is obtained over the entire frequency range.

In theory, one could approximate the frequency response up to very large frequencies using low-frequency samples and a large number of higher-order derivatives. In practice, the first-order derivatives turn out to be the most useful ones. Since the behavior of the frequency response is relatively smooth, the higher-order derivatives ( $k > 2$ ) become very small, and may drop below machine precision.

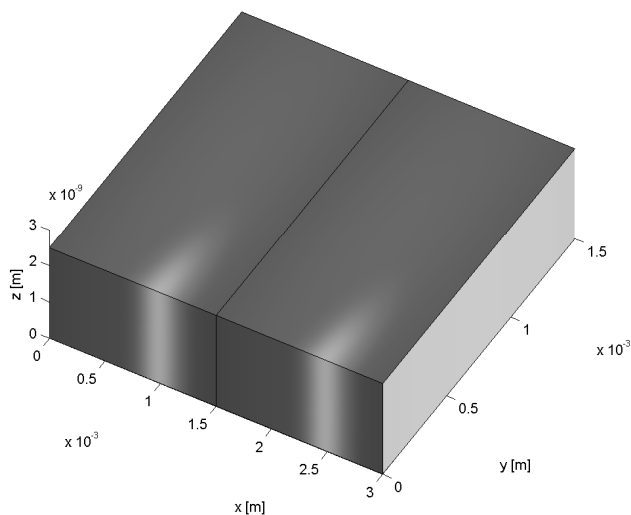


Figure 1: Two touching cells

## 5 Conclusions

Numerical integration of partial elements using a Gauss-Legendre quadrature can be computationally expensive, and requires a large number of integration points to obtain accurate results beyond the extended frequency range. It is shown that a good overall approximation can be obtained if a model is built using low-frequency samples and their first-order derivatives only. Since frequency derivatives can be computed using approximately the same order of integration as the corresponding low-frequency sample, the overall macromodeling time can be reduced.

## 6 Acknowledgements

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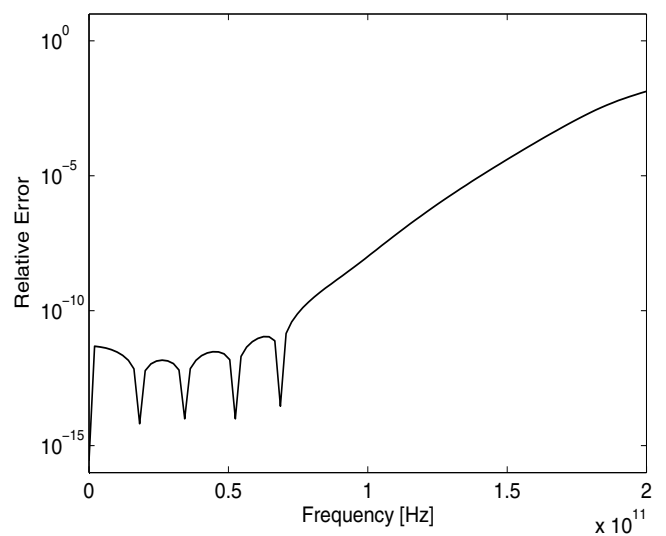
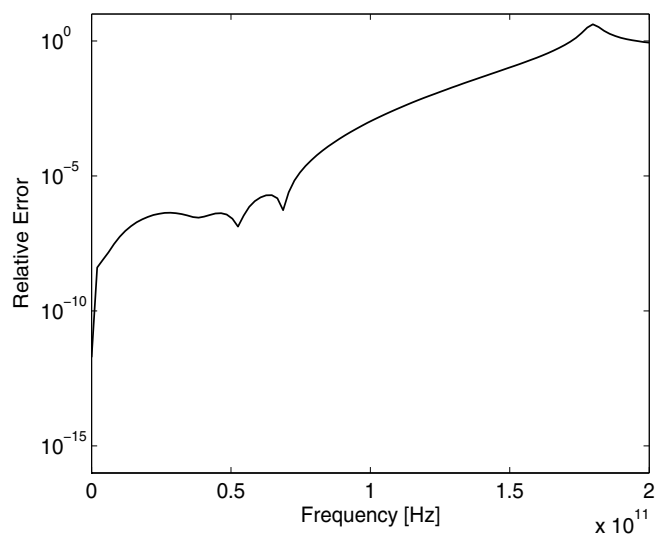
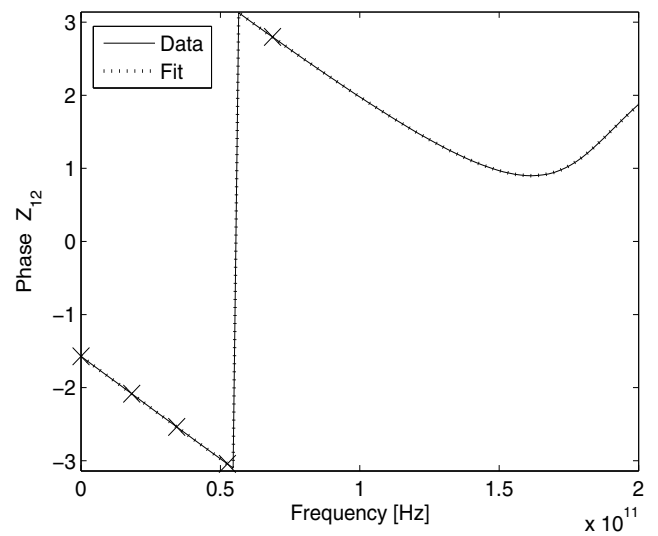
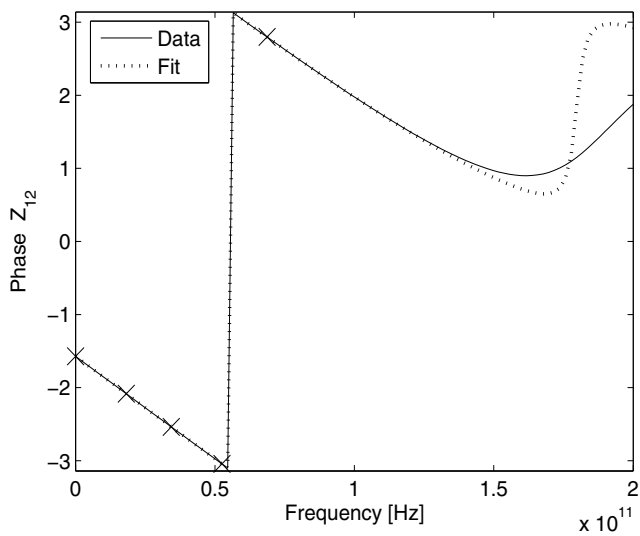
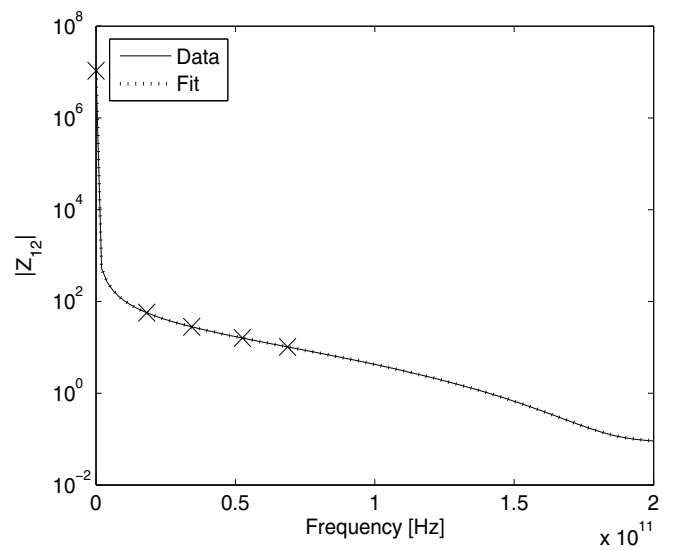
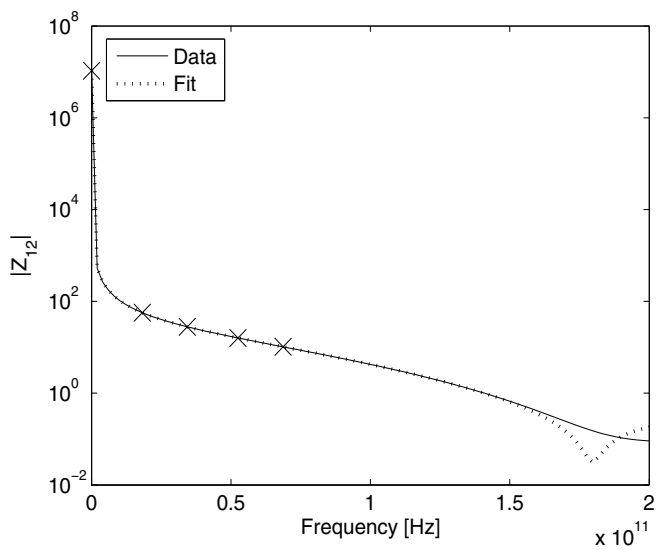


Figure 2: Magnitude, phase and error of rational fitting model using 5 samples without higher order derivatives

Figure 3: Magnitude, phase and error of rational fitting model using 5 samples including 1st order derivatives